+CMYK+

Secondary

MATHEMATICS

Class-VI

Publication Division

D.A.V. College Managing Committee

Chitra Gupta Road, New Delhi-110055

+CMYK+

Contents

CHAPTER 1 NATURAL NUMBERS AND WHOLE NUMBERS

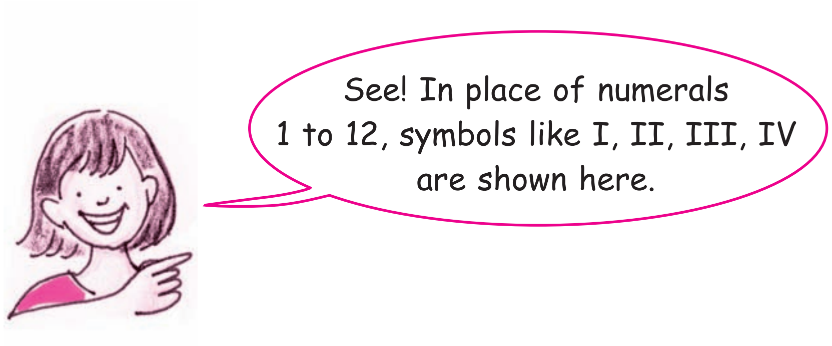
INTRODUCTION

Do you remember numbers? Let us solve some problems.

1. **Fill in the following blanks.**
   (a) The place value of 5 in 37572 is ________ .
   (b) 8 occurs at ________ place in 105876.
   (c) Place value of 4 in 42160 is ________ .
   (d) 5 occupies the ________ place in 37652.
   (e) The face value of 7 in 4709606 is ________ .
   (f) $3 \times 100000 + 5 \times 1000 + 7 \times 10 + 8 \times 1 =$ ________ .
   (g) $200000 + 4000 + 800 + 6 =$ ________ .
2. **Find the product of the place value and face value of 5 in 76085432.**
3. **Find the product of the largest 4-digit number and the smallest 4-digit number. Write the product in expanded form also.**
4. **Write all the possible 3-digit numbers using the digits 7, 5, 1.**
   (Repetition not allowed)
5. **Write all the possible 3-digit numbers using the digits 4, 0, 6.**
   (Repetition not allowed)
6. **Write the following numbers in Indian System of Numeration.**
   (a) 8751432 (b) 60002 (c) 491603 (d) 632245687
7. **Write the following numbers in International System of Numeration.**
   (a) 5737802 (b) 411809 (c) 33246951 (d) 898576449
8. **Write the numerals for the following:**
   (a) Thirty two million four thousand three hundred and twenty nine.
   (b) Thirty nine crore forty eight lakh nine thousand and eighty eight.
9. **How many lakhs make 6 millions?**
10. **How many millions make 17 crores?**

ROMAN NUMERALS

Have you ever seen a clock of this type?



These symbols are called **Roman Numerals.**

Now observe these Hindu Arabic Numerals and their corresponding Roman Numerals.

Hindu Arabic Numerals	I	5	10	50	100	500	1000
Roman Numerals	I	V	X	L	C	D	M

The rules for this system of numeration are given below:

- **Rule 1 –** If a symbol is repeated, its value is added as many times as it occurs.

  For example: II = 1 + 1 = 2

  XXX = 10 + 10 + 10 = 30

- **Rule 2 –** A symbol is not repeated more than three times but the symbols V, L and D are never repeated.

- **Rule 3 –** If a symbol of smaller value is written to the right of a symbol of greater value, its value gets added to the value of greater symbol.

  For example: VI = 5 + 1

  = 6

  LXV = 50 + 10 + 5

  = 65

- **Rule 4 –** If a symbol of smaller value is written to the left of a symbol of greater value, its value is subtracted from the symbol of the greater value.

  For example: IV = 5 – 1 = 4

  XL = 50 – 10 = 40

  XC = 100 – 10 = 90

- **Rule 5 –** The symbols V, L and D are never written to the left of a symbol of greater value, i.e. V, L, D are never subtracted.

Observe the Roman Numerals corresponding to some Hindu Arabic Numerals.

1 = I	10 = X
2 = II	20 = XX
3 = III	30 = XXX
4 = IV	40 = XL
5 = V	50 = L
6 = VI	60 = LX
7 = VII	70 = LXX
8 = VIII	80 = LXXX
9 = IX	90 = XC
10 = X	100 = C

Let us study some examples.

Example 1: Write the Roman Numerals corresponding to the following Hindu Arabic Numerals.

(a) 19 (b) 56 (c) 44 (d) 98 (e) 78

Solution:

(a) 19 = 10 + 9
= XIX

(b) 56 = 50 + 6
= LVI

(c) 44 = 40 + 4
= XLIV

(d) 98 = 90 + 8
= XCVIII

(e) 78 = 70 + 8
= (50 + 10 + 10) + 8
= LXXVIII

Example 2: Convert the following into Hindu Arabic Numerals.

(a) LXXIX (b) XLIX (c) XCVII (d) XCI

Solution:

(a) LXXIX = 50 + 10 + 10 + 9
= 79

(b) XLIX = 40 + 9
= 49

(c) XCVII = 90 + 7
= 97

(d) XCI = 90 + 1
= 91

Worksheet 1

1. Write the Roman Numeral for each of the following:

(a) 33	(b) 500	(c) 48	(d) 76	(e) 95
(f) 41	(g) 87	(h) 66	(i) 19	(j) 1000

2. Convert the following into Hindu Arabic Numerals.

(a) XXVI	(b) LXXVII	(c) XCI	(d) LXXXV	(e) D
(f) XCIX	(g) XCVII	(h) LV	(i) XLI	(j) XXIX

3. Solve and write the results in Roman Numerals.

(a) $32 + 67$	(b) $216 - 174$
(c) 12×7	(d) $3645 \div 45$

4. Which of the following is meaningless?

(a) VVII	(b) XLI	(c) LIV	(d) IC	(e) LIL
(f) IVC	(g) XCI	(h) VL		

5. Match the following:

DXLV	908
MMX	591
CMVIII	545
CCIII	2010
DXCI	203

6. Write the following in Roman Numerals.

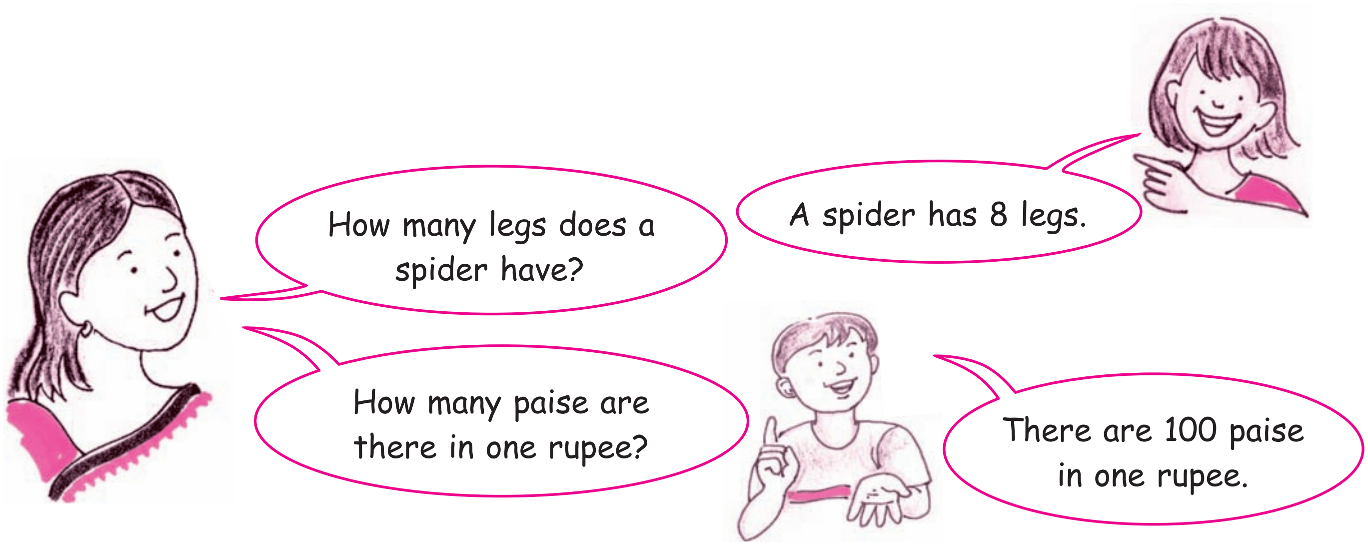
(a) Year in which India got Independence.

(b) Year in which India became Republic.

(c) Year in which you were born.

(d) Present year.

WHOLE NUMBERS AND THEIR REPRESENTATION ON NUMBER LINE



So we have used the numbers 1, 2, 3, 4, ... for answering these questions.

Numbers 1, 2, 3, 4, ... which we use for counting form the system of **Natural Numbers** (Counting numbers).

Remember

- The smallest natural number is 1.
- We cannot find the greatest natural number.

Look at the following picture. What is the number of boys in this group?

The number of boys in this group is zero (0).

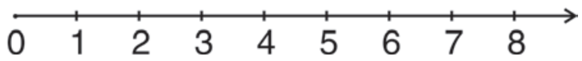
Natural numbers along with zero form the system of **Whole Numbers**.

Remember

- The smallest whole number is 0.
- We cannot find the greatest whole number.

For the teacher:
Explain to the students that these numbers are equidistant on the number line.

Now look at the whole numbers given on a number line.



SUCCESSOR AND PREDECESSOR

One more than any whole number is called the **successor** of that whole number.

For example: 51 is the successor of 50

10000 is the successor of 9999

Number to the Right

One less than any whole number is called the **predecessor** of that whole number.

For example: 61 is the predecessor of 62

99999 is the predecessor of 100000

Number to the Left

Let us take up some examples.

Example 3: Write the greatest 4-digit number using the digits 5, 0, 2. **(digits may repeat)**

Solution: Any 4-digit number occupies four places, i.e. thousands, hundreds, tens and ones. Since 5 is the largest number here, it will occupy most of the places in the required number and rest of the numbers will occur only once and that too in descending order. So, the required number will be,

Th	H	T	O
5	5	2	0

Example 4: Rearrange the digits of 72094186 to form the smallest 8-digit number.

Solution: We write the digits in ascending order–

0, 1, 2, 4, 6, 7, 8, 9

Since we cannot start a number with zero, we start the number with 1. So the required number is–

1, 02, 46, 789

Worksheet 2

1. Complete the statements by filling in the blanks.

(a) The smallest whole number is _______.

(b) There is _______ largest whole number.

(c) In whole numbers, _______ has no predecessor.

(d) The predecessor of the smallest 5-digit number has _______ digits.

(e) The successor of the greatest 5-digit number is ________________.

(f) The smallest 7-digit number ending in 5 is __________________.

(g) 387 is to the _______ of 388 on the number line.

(h) 4397 is to the _______ of 4396 on the number line.

2. Write the successor of the following:

(a) 45638 (b) 10009 (c) 220209 (d) 4226372

3. Write the predecessor of the following:

(a) 33801 (b) 100000 (c) 6698979 (d) 80115670

4. Find the next three successors of 647999.

5. Find the three immediate predecessors of 552002.

6. Compare the following numbers:

(a) 729 ◯ 279 (b) 10899 ◯ 10799

(c) 9785 ◯ 7835 (d) 135629 ◯ 136529

7. Arrange the following in ascending order.

43, 287, 15769, 833, 49538, 34, 798665

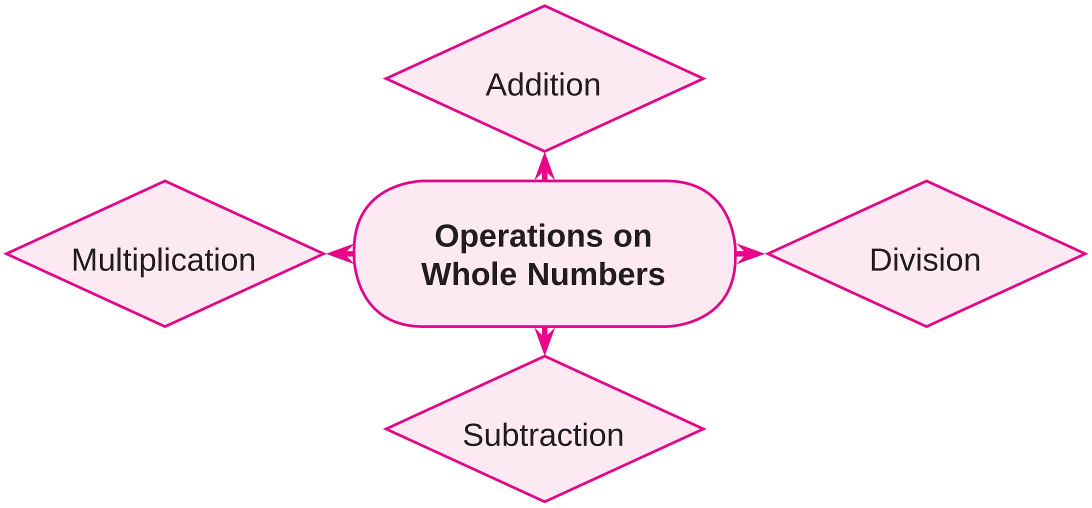
8. Arrange the following in descending order.

3951, 1024, 977, 422596, 38675, 560832, 67.

9. Form the greatest 7-digit number using the digits 3, 8, 9.
(digits may repeat)

10. Write the smallest 6-digit number using the digits 4, 5, 0.
(digits may repeat)

OPERATIONS ON WHOLE NUMBERS



Let us take up the properties of each and every operation one by one.

A. ADDITION OF WHOLE NUMBERS

Properties of Addition

Property-1: The sum of two whole numbers is again a whole number.

e.g. 3 + 8 = 11

Property-2: The sum remains the same even after changing the order of addends.

e.g. 23 + 18 = 18 + 23

Property-3: The sum remains the same, when the order or the grouping of three or more addends is changed.

e.g. 11 + (18 + 25) = (11 + 18) + 28

Property-4: When a number is added to zero or zero is added to the number, sum is the number itself.

e.g. 7 + 0 = 0 + 7 = 7

Let us take up an example to see that the sum remains same even if the order of the addends is changed.

Example 5: Add 469, 35, 31, 5 in 2 different ways.

Solution:

469 + 35 + 5 + 31
504 + 36
540

469 + 31 + 35 + 5
500 + 40
540

(By interchanging the order of the addends.)

B. SUBTRACTION OF WHOLE NUMBERS

Properties of Subtraction

Property-1: The difference between two whole numbers may or may not be a whole number. e.g. 5 – 4 = 1 (is a whole number). But 4 – 5 = – 1 is not a whole number.

Property-2: The difference between two same whole numbers is always zero.

e.g. 5 – 5 = 0

Property-3: For any three whole numbers, say 6, 4, 2 (6 – 4) – 2 is not equal to 6 – (4 – 2).

Property-4: When zero is subtracted from a whole number, the difference is the number itself.

e.g. 5 – 0 = 5

Property-5: If 8, 5, 3 are whole numbers, such that 8 – 5 = 3 then 5 + 3 = 8

Let us study an example based on Property-5.

Example 6: Subtract 40 from 96

Solution: 96 – 40 = 56

or 96 = 56 + 40

Worksheet 3

1. **Fill in the blanks to make the following statements true.**

(a) 1794 + 624 = 624 + ________

(b) (287 + 163) + 800 = 287 + (________ + 800)

(c) 432 + ________ = 111 + 432

(d) 97 + 561 = ________ + 97

(e) (200 + 1020) + 3303 = ________ + (200 + ________)

(f) 0 + 268 = ________

(g) 469 – 0 = ________

(h) 1238 – ________ = 1238

(i) 29487 + ________ = 29487

2. **Replace (*) with the appropriate digit.**

(a)
$$\begin{array}{r} 2\,9\,4\,2\,2 \\ -\quad 6\,8\,*\,5 \\ \hline 2\,*\,5\,4\,7 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 4\,7\,8\,*\,5 \\ +\,3\,*\,3\,3\,4 \\ \hline 8\,4\,*\,3\,9 \\ \hline \end{array}$$

(c)
$$\begin{array}{r} 2\,3\,9\,*\,8 \\ +\,1\,*\,9\,8\,0\,* \\ \hline 1\,4\,3\,*\,6\,9 \\ \hline \end{array}$$

(d)
$$\begin{array}{r} 8\,0\,0\,1\,9 \\ -\,*\,4\,3\,*\,* \\ \hline 2\,5\,*\,0\,3 \\ \hline \end{array}$$

3. **Add 718662 to 360895. Now add 360895 to 718662. Are the two results same?**

4. **Add the following numbers by rearranging them: (Use property here)**

(a) 786 + 342 + 214

(b) 479 + 2000 + 21

(c) 225 + 725 + 275 + 275

(d) 67 + 1376 + 624 + 933

(e) 637 + 908 + 363

(f) 2062 + 547 + 938 + 353

5. **Subtract the following and check your answer by corresponding addition.**

(a) 29435 – 17005

(b) 100000 – 62581

(c) 75691 – 45512

(d) 77426 – 71236

6. **In a school, the number of students is 5637. If 142 students took admission during that year, find the total number of students in the school.**

7. **The price of a car is ₹ 3,76,866. If it is increased by ₹ 42,049, find the new price of the car.**

8. **A club organises a trip to the Disney World. The cost of the whole package is ₹ 1,83,420. The club gives a discount of ₹ 47,632. What is the cost of the package after the discount?**

9. **Rahul deposited ₹ 57,630 in the bank. After a week, he withdrew ₹ 19,211. What is the current balance in Rahul's account?**

10. **A garment factory produces 33000 trousers every year. Out of these, 12309 are for men and 9538 are for women. Find the number of trousers produced for children.**

C. MULTIPLICATION OF WHOLE NUMBERS

Properties of Multiplication

Property-1: If two whole numbers are multiplied in either order, the product remains the same.

e.g. $3 \times 8 = 8 \times 3 = 24$

Property-2: If three numbers are multiplied in any grouping or order, the product remains the same.

e.g. $2 \times (5 \times 7) = (2 \times 5) \times 7 = (2 \times 7) \times 5 = 70$

Property-3: The product of a whole number and 1 is the number itself.

e.g. $1 \times 5 = 5 \times 1 = 5$

Property-4: The product of any whole number and zero is zero.

e.g. $2 \times 0 = 0 \times 2 = 0$

Worksheet 4

1. **Use the properties of multiplication and fill in the following blanks.**

   (a) $0 \times 489 =$ ________

   (b) $1 \times 741 =$ ________

   (c) $27 \times 635 = 635 \times$ ________

   (d) $(242 \times 197) \times 581 = 242 \times (197 \times$ ________ $)$

   (e) $479 \times$ ________ $= 479$

   (f) ________ $\times 831 = 0$

   (g) $162 \times 0 \times 1025 =$ ________

2. **If the cost of one burger is ₹ 50.50, what will be the cost of 25 such burgers?**

3. **In a library, there are 27 book shelves. If there are 479 books on each book shelf, find the total number of books in the library.**

4. **A store has 432 dresses for girls. If the cost of each dress is ₹ 583.50, find the cost of all dresses.**

MORE ABOUT MULTIPLICATION PROPERTIES

Consider the numbers 3, 4 and 5.

Let us add 3 and 4 and multiply the sum by 5

$$(3 + 4) \times 5$$
$$\downarrow$$
$$7 \times 5$$
$$= 35$$

Now multiply 3 and 4 separately by 5 and then add the products.

$$3 \times 5 + 4 \times 5$$
$$15 + 20$$
$$35$$

In both the cases, we get 35. So, we can say that–

$(3 + 4) \times 5 = 3 \times 5 + 4 \times 5$

Similarly, $(7 - 3) \times 5 = 7 \times 5 - 3 \times 5$

> This is known as **Distributive Property of Multiplication.** It is useful for multiplying large numbers.

Example 7: Multiply 172×97

Solution: We know that $97 = (100 - 3)$

$172 \times (100 - 3)$

or $172 \times (100 - 3) = 172 \times 100 - 172 \times 3$

$= 17200 - 516$

$= 16684$

Example 8: Solve $569 \times 45 + 569 \times 55$

Solution: $569 \times (45 + 55)$ ← Taking out 569 as common factor from both the products.

$= 569 \times 100$ ← Multiplying by 100 orally.

$= 56900$

Example 9: Solve $361 \times 162 - 361 \times 60 - 2 \times 361$

Solution: $361 \times (162 - 60 - 2)$ ← Taking out 361 as common factor and putting rest of the terms in a bracket.

$= 361 \times (162 - 62)$

$= 361 \times 100$

$= 36100$

Remember

By rearranging the order of the numbers, multiplication becomes easy. We try to combine the numbers that produce maximum number of zeroes after the multiplication.

Worksheet 5

1. **Fill in the following blanks by using different properties of multiplication.**

   (a) 52 × (63 + 37) = (52 × ______) + (______ × 37)

   (b) 297 × (______ + 43) = (297 × 88) + (297 × ______)

   (c) ______ × (84 + 16) = 36 × 84 + 36 × ______

   (d) 218 × 94 = (218 × ______) – (218 × 6)

   (e) 778 × 994 = (778 × 1000) – (778 × ______) – 778.

2. **Rearrange the numbers and then multiply them.**

   (a) 125 × 488 × 8
   (b) 625 × 25 × 20 × 4
   (c) 16 × 125 × 8 × 625
   (d) 20 × 1975 × 5
   (e) 8 × 25 × 125 × 40
   (f) 200 × 625 × 16 × 50

3. **Find the product by using distributive property.**

   (a) 241 × 107
   (b) 685 × 94
   (c) 439 × 995
   (d) 1009 × 1392
   (e) 98 × 553
   (f) 999 × 399

4. **Find the value by using distributive property.**

   (a) 1562 × 62 + 1562 × 38
   (b) 638 × 176 – 638 × 75 – 638
   (c) 85 × 15 + 15 × 15
   (d) 688 × 10 × 437 – 6880 × 337
   (e) 125 × 8 × 883 + 117 × 25 × 40
   (f) 750 × 17 + 750 × 38 + 27 × 750 + 18 × 750

5. **Rohan buys 12 computers and 12 printers. If the cost of one computer and one printer is ₹ 56,233 and ₹ 7,867 respectively, find the total cost incurred by Rohan.** **(Use distributive property of multiplication.)**

6. **In a school, the monthly fee of a child is ₹ 497. If there are 2983 students in a school, find the total fee collected in a month.**

(Use distributive property of multiplication.)

D. DIVISION OF WHOLE NUMBERS

Property-1: If two whole numbers are divided, their quotient may or may not be a whole number.

e.g. $3 \div 6 = \frac{1}{2}$ but $6 \div 3 = 2$

Property-2: A number divided by itself, gives the quotient as 1.

e.g. $5 \div 5 = 1$.

Property-3: A number divided by one gives the quotient as the number itself.

e.g. $4 \div 1 = 4$

Property-4: A multiplication fact of two distinct and non-zero whole numbers gives two division facts.

e.g. $4 \times 5 = 20$ and $20 \div 5 = 4$, $20 \div 4 = 5$

Property-5: Zero divided by any number gives the quotient as zero.

e.g. $0 \div 3 = 0$

We also know

In division

Dividend = Divisor × Quotient + Remainder

Let us take up some examples.

Example 10: Find the least number that should be subtracted from 1000 so that 30 divides the difference exactly.

Solution: Divide 1000 by 30

$$\begin{array}{r|l|l} 30 & 1000 & 33 \\ & \underline{90} & \\ & 100 & \\ & \underline{90} & \\ & \underline{10} & \end{array}$$

$1000 - 10 = 990$

So, 10 should be subtracted from 1000 so that the difference, i.e. 990 is exactly divisible by 30.

Example 11: Find the least number that should be added to 1000 so that 35 divides the sum exactly.

Solution:

$$
\begin{array}{r|r|l}
35 & 1000 & 24 \\
 & 70 & \\ \cline{2-2}
 & 300 & \\
 & 280 & \\ \cline{2-2}
 & 20 &
\end{array}
$$

The difference between divisor and remainder is 35 – 20 = 15

Therefore, 15 should be added to 1000 so that the sum 1015 is exactly divisible by 35.

Worksheet 6

1. **Divide and check your answer.**

   (a) 2781 ÷ 35   (b) 49277 ÷ 511

   (c) 7335 ÷ 122   (d) 64895 ÷ 247

2. **Find the least number that should be subtracted from 1000 so that 35 divides the difference exactly.**

3. **Find the least number that should be added to 2000 so that 45 divides the sum exactly.**

4. **Find the largest 5-digit number which is exactly divisible by 40.**

5. **In a parade, the soldiers are arranged in 14 rows. If the number of soldiers is 504, find the number of soldiers in each row.**

6. **In a dance class, 137 students got themselves enrolled. If the total fee collected is ₹ 3,56,200, find the fee paid by each student.**

ESTIMATION

Do you remember Rounding off numbers? Let us recall.

1. **Round off the given numbers as directed.**

   (a) 48 (to the nearest ten)

   (b) 3,285 (to the nearest thousand)

   (c) 87,08,463 (to the nearest ten lakh)

   (d) 4,53,73,043 (to the nearest crore)

2. **Round off the given numbers as directed.**

   (a) 3.84 (to the nearest ones)

   (b) 21.472 (to the nearest hundredth)

   (c) 1.53 (to the nearest tenth)

3. **Round 4,25,163 to the nearest hundred, ten thousand and lakh.**

ESTIMATION OF OUTCOMES OF NUMBER SITUATIONS

Let us take some situations.

Situation 1: Rohan plans to give a treat to his eight friends in school on his birthday. His father gave him ₹ 500 for this. He decides to give a sandwich, pastry and fruit juice to these friends. One sandwich costs ₹ 20, one pastry costs ₹ 25 and one fruit juice costs ₹ 15. Rohan roughly calculates the amount he needed. This will be the sum of amount he spends on these three items.

Situation 2: On a particular day a businessman has to receive ₹ 5,38,485 and ₹ 2,19,560 from two different parties. He also has to pay a sum of ₹ 6,35,750 to someone on the same day. He quickly round off the numbers to the nearest lakh and then works out if he will be able to pay the money by evening. Will he be able to pay back the amount?

The estimation of outcomes of numbers is a reasonable guess of the actual value.

Remember

- **Estimating** means approximating a quantity to the accuracy required.
- Estimation is done by rounding off the numbers involved and getting a quick and rough answer.

ESTIMATION OF SUM OR DIFFERENCE

When we estimate sum or difference, we should have an idea of why we need to round off and therefore, the place to which the rounding is needed.

Example 12: Estimate 4,356 + 13,849

Solution: We shall round off the numbers to the nearest thousands.

13,849 is rounded off to 14,000

4,356 is rounded off to 4,000

Estimated sum = 14,000 + 4,000

= 18,000

Example 13: Estimate 7,412 – 236

Solution: Let us round off these numbers to the nearest thousands.

7,412 is rounded off to 7,000

236 is rounded off to 0

Estimated difference = 7000 – 0

= 7000

This is not a reasonable estimate. Why?

We need a closer estimate.

Let us round the numbers to the nearest hundreds.

7,412 is rounded off to 7,400

236 is rounded off to 200

Estimated difference = 7,400 – 200

= 7,200

This is a better and more meaningful estimate.

Worksheet 7

Estimate.

1. 215 + 436
2. 1,238 + 4,298
3. 15,409 + 3,288
4. 618 + 561 + 372
5. 869 – 341
6. 8,565 – 4,341
7. 1,048 – 692
8. 78,432 – 71,496

ESTIMATE OF PRODUCT OF NUMBERS

Let us estimate 63 × 182

If we round off 63 to the nearest hundred, we get 100

If we round off 182 to the nearest hundred, we get 200

Hence, the estimated product = 100 × 200 = 20,000

This is much greater than the actual product.

So to get a more reasonable estimate, we try rounding off 63 to the nearest tens that is 60, and also 182 to the nearest tens that is 180.

We get $60 \times 180 = 10800$

This is a good estimate but not quick enough.

So we round off 63 to the nearest ten which is 60 and 182 to the nearest hundred which is 200.

Now the estimated value of $63 \times 182 = 60 \times 200$

$= 12,000$

12,000 is a quick and good estimate of the product of numbers.

Remember

To estimate the product, round off each factor to its greatest place, then multiply the rounded off factors.

Example 14: Estimate 52×786

Solution: 52 can be rounded off to the nearest ten as 50.

786 can be rounded off to the nearest hundred as 800.

Hence, the estimate product $= 50 \times 800 = 40,000$

Worksheet 8

Estimate the given products.

1. 61×47
2. 589×245
3. 9×677
4. 864×342
5. 913×752
6. $4,329 \times 609$
7. $1,234 \times 5,678$
8. $13,459 \times 7,801$

BRACKETS AND THEIR USE

Do you remember solving numerical expressions involving the fundamental operations of addition, subtraction, multiplication and division?

Recall the DMAS Rule—

Division	$\rightarrow$	First
Multiplication	$\rightarrow$	Second
Addition	$\rightarrow$	Third
Subtraction	$\rightarrow$	Last

Use this rule to simplify:

1. $3 + 6 \div 2 - 4$
2. $49 \div 7 + 7 \times 2$
3. $1\frac{1}{2} + \frac{3}{4} \times \frac{4}{5} - \frac{1}{5}$
4. $3.5 - 0.1 \times 5 + 1.2$

Let us now learn to solve numerical expressions involving brackets. Most commonly used brackets are:

Brackets symbol	Name
( )	Parentheses or Round brackets
{ }	Curly brackets
[ ]	Square brackets

In writing mathematical expressions consisting of more than one brackets, Round brackets are used in the innermost part followed by Curly brackets and these two are covered by Square brackets.

We first perform the operations within the Round brackets followed by the operations within the Curly brackets and lastly within the Square brackets.

Example 15: Simplify $27 - [5 + \{28 - (17 - 7)\}]$

Solution: We have $27 - [5 + \{28 - (17 - 7)\}]$

$= 27 - [5 + \{28 - 10\}]$ ⟵ Removing Round brackets

$= 27 - [5 + 18]$ ⟵ Removing Curly brackets

$= 27 - 23$ ⟵ Removing Square brackets

$= 4$

Example 16: Simplify 45 – [38 – {60 ÷ 3 – (9 – 7 + 3)}]

Solution: We have 45 – [38 – {60 ÷ 3 – (9 – 7 + 3)}]

= 45 – [38 – {60 ÷ 3 – 5}]

= 45 – [38 – {20 – 5}]

= 45 – [38 – 15]

= 45 – 23

= 22

Worksheet 9

Simplify the following numerical expressions.

1. 25 + 14 ÷ (5 – 3)
2. 3 – (5 – 6 ÷ 3)
3. 36 – [12 + (3 × 10 ÷ 2)]
4. 20 – 3 – [7 – {2 + (4 – 3)}]
5. 15 + [18 – {4 + (16 – 5)}]
6. $22 - \frac{1}{4}\{16 - (8 \div 4 + 2)\}$
7. 18 – [18 – {18 – (18 – 18) – 18}]
8. 150 – [70 – {60 – (30 + 20)} – 10]

VALUE BASED QUESTIONS

1. **Members of an NGO decided to provide blankets to an old age home. For this purpose a sum of ₹ 8435 was collected and 35 blankets were purchased. The old people were very happy with the blankets. They blessed the NGO members for their concern for the old people.**
   (a) What is the cost of one blanket donated?
   (b) Name any two items that you can donate to an old age home.
2. **Trees not only make the air pure but also beautify the environment. In a school, the members of Eco club were taken for a trip to a nearby nursery. As a part of a project the children planted 95 saplings of different trees in the nursery. The cost of each sapling was ₹ 175. The children were very thrilled and happy with the project.**
   (a) What is the amount spent on the saplings? (Use distributive property)
   (b) Name any two saplings that you will like to plant in your garden or nearby park.

BRAIN TEASERS

1. **A. Tick (✓) the correct answer.**

(a) Which of the following is meaningless?

(i) XLVI (ii) ICVII (iii) XML (iv) XLIX

(b) The greatest 2-digit number exactly divisible by 17 is–

(i) 68 (ii) 91 (iii) 85 (iv) 97

(c) The smallest 5-digit number formed by using the digits 3, 0, 1 (Repetition of digits allowed) is–

(i) 10003 (ii) 10013 (iii) 13000 (iv) 00013

(d) The estimated value of $36 + 71 - 55$ is–

(i) 40 (ii) 50 (iii) 70 (iv) 150

(e) Which of the following is not a natural number?

(i) $3 + 5 - 2$ (ii) 4×0 (iii) $8 \div 8$ (iv) $6 - 3 + 1$

B. Answer the following questions.

(a) How many millions make 3 crores?

(b) Which whole number does not have a successor?

(c) What is the estimated value of 786×1385?

(d) What is the value of $125 \times 4 \times 25 \times 8$?

(e) What is the difference between the place value and face value of 8 in 38,46,197?

2. **Write the greatest 6-digit number using three different digits.**

3. **Find the smallest and greatest 7-digit and 8-digit numbers using the digits 5, 0, 4, 1.**

4. **Find the difference between the largest and the smallest 7-digit numbers formed by using the digits in the number 6427310. (digits should not repeat)**

5. **Using distributive property, simplify:**

$223 \times 25 \times 6 - 223 \times 10 \times 15$

6. **Complete the series–**

1, 1, 2, 3, 5, 8, 13, 21, 34, ______, ______,

7. **Fill in the blanks in the following magic square.**

	10	3	6
4	5		9
	11	2	7
1	8		

8. **Form the greatest 6-digit number using the digits of prime numbers between 80 and 100.**

9. **Find the number which is–**

   (a) the successor of the successor of 304998.

   (b) the predecessor of the predecessor of the smallest 6-digit number.

10. **Fill in the blanks using Roman Numerals.**

    (a) CIXX – $\boxed{}$ = XXV (b) $\boxed{}$ + XLVI = LXX

11. **Arrange the following in ascending order.**

    LVII, XC, XV, LXIV, LXXI, XXIX

12. **Estimate the following:**

    (a) 234 + 649 – 186 (b) 9483 – 6321 – 2178

    (c) 3284 × 639 (d) 12345 × 6789

HOTS

1. **How many times does the digit 7 occurs if we write all the numbers from 1 to 200?**

2. **Write all the 2-digit numbers which when added to 27 get reversed.**

ENRICHMENT QUESTIONS

1. **Get 100 using four 9's and some of the symbols like +, –, ×, ÷**

2. **A number is three times the sum of its digits. Find the number.**

YOU MUST KNOW

1. Various systems of numerations are used in different parts of the world. We use the Hindu–Arabic System of Numeration. Another systems of writing numerals is called Roman System.
2. The numbers 1, 2, 3, ... which we use for counting are called Counting numbers or Natural numbers. The numbers 0, 1, 2, 3, ... form the set of Whole numbers. All natural numbers are whole numbers but all whole numbers are not natural numbers.
3. Every whole number can be represented on the number line. Every whole number has a successor. Every whole number except zero has a predecessor.
4. Addition of two whole numbers always give a whole number. Similarly multiplication of two whole numbers is always a whole number. But this is not true for the operations of subtraction and division.
5. Zero is the identity element of addition and one is the identity element of multiplication.
6. The sum remains the same if the order or group of three or more addends is changed. Similarly when three or more numbers are multiplied the product remains the same.
7. Multiplication is distributive over addition for whole numbers.
8. In division, Dividend = Divisor × Quotient + Remainder
9. There are number of situations in which we do not need the exact number of quantity but only a reasonable guess or estimation. Estimation involves approximating a quantity to an accuracy required.
10. In some situations, we need to estimate the outcome of number operations. A quick rough answer is obtained by rounding off the numbers involved in the operation.

CHAPTER 2

FACTORS AND MULTIPLES

INTRODUCTION

Do you remember factors and multiples? Let us recall them once again.

Multiples: For getting multiples of a number, we recite the multiplication table of that number.
e.g. multiples of 4 are 4, 8, 12, 16 .....

Factors: A factor of a number divides the number exactly (with zero as the remainder).
e.g. the factors of 12 are 1, 2, 3, 4, 6, 12.

Prime Number: A number which has only two different factors, 1 and the number itself is called prime number.
e.g. 2, 3, 5, 7 .....

Composite Number: A number which has more than two different factors is called composite number.
e.g. 4, 6, 8, 9 .....

MORE ABOUT FACTORS

One (1) is a factor of every number.

Every number is a factor of itself.

Two prime numbers whose difference is 2 are called **Twin Prime Numbers.**
e.g. 5 and 3; 41 and 43.

Two numbers are said to be **Co-prime** when they have only 1 as common factor.
e.g. 3 and 5; 19 and 20.

Example 1: State whether the following are prime or composite by listing their factors:

(a) 36 (b) 13

Solution: (a) 36

We have $1 \times 36 = 36$

$2 \times 18 = 36$

$$3 \times 12 = 36$$
$$4 \times 9 = 36$$
$$6 \times 6 = 36$$

Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36.

Therefore, 36 is a composite number.

(b) 13

We have $1 \times 13 = 13$

Factors of 13 are 1 and 13.

Therefore, 13 is a prime number.

Example 2: List the first five multiples of 19.

Solution: The required multiples are–

$$1 \times 19 = 19$$
$$2 \times 19 = 38$$
$$3 \times 19 = 57$$
$$4 \times 19 = 76$$
$$5 \times 19 = 95$$

19, 38, 57, 76 and 95 are the first five multiples of 19.

Worksheet 1

1. Fill in the following blanks.

(a) Numbers which have more than two different factors are called ______________ .

(b) Numbers which are not divisible by any other number except 1 and the number itself are called ______________ .

(c) 1 is neither ______________ nor composite.

(d) 6 is a composite number as it has ______________ factors.

(e) ______________ is the only even prime number.

(f) The smallest prime number is ______________ .

(g) The smallest composite number is ______________ .

(h) The smallest odd composite number is ______________ .

(i) The greatest 2-digit prime number is ______________ .

2. **Are the following numbers prime or composite. Show by finding the factors.**

   (a) 9 (b) 48 (c) 89 (d) 96 (e) 78 (f) 101

3. **Write down the first ten prime numbers.**

4. **Write down all the prime numbers between 50 to 110.**

5. **A number lies between 2000 and 2070 and has 5 in its ones place. Is it a prime or composite number? Give reasons.**

6. **List the first five multiples of–**

   (a) 25 (b) 17 (c) 100 (d) 41

7. **List all the multiples of 15 between 50 to 100.**

8. **Between which multiples of 10 does 3486 lie?**

9. **Write any four pairs of twin primes.**

10. **Which of the following numbers are co-prime?**

    (a) 13, 14 (b) 8, 20 (c) 31, 59 (d) 34, 85

TEST OF DIVISIBILITY

There are certain tests which can confirm whether a number is divisible by some other number. Given below are the tests of divisibility.

I. Divisibility by 2

Is 368 divisible by 2? (Yes/No)

Is 490 divisible by 2? (Yes/No)

Is 43 divisible by 2? (Yes/No)

Is 48 divisible by 2? (Yes/No)

Here we see that 368, 490 and 48 are divisible by 2 whereas 43 is not divisible by 2.

A number is divisible by 2 if the digit at ones place is divisible by 2, i.e. if the digit at ones place is 0, 2, 4, 6 or 8.

II. Divisibility by 5

Is 8955 divisible by 5? (Yes/No)

Is 6320 divisible by 5? (Yes/No)

Is 7939 divisible by 5? (Yes/No)

Is 387 divisible by 5? (Yes/No)

Here we can see that 8955 and 6320 are divisible by 5 but 7939 and 387 are not divisible by 5.

A number is divisible by 5 if the digit at ones place is 0 or 5.

III. Divisibility by 10

Is 7442 divisible by 10? (Yes/No)

Is 10240 divisible by 10? (Yes/No)

Is 73 divisible by 10? (Yes/No)

Is 1390 divisible by 10? (Yes/No)

Here the numbers 10240 and 1390 are divisible by 10 but 7442 and 73 are not divisible by 10.

A number is divisible by 10 if the digit at ones place is 0.

IV. Divisibility by 4

Is 6943284 divisible by 4?

Step 1: Separate the number formed by the digits at tens and ones place.

69432 / 84

Step 2: Now divide 84 by 4.

$$\begin{array}{r|l} & 21 \\ 4 & 84 \\ & 8 \\ \hline & \ 4 \\ & \ 4 \\ \hline & \ 0 \end{array}$$

84 is divisible by 4.

Hence, 6943284 is also divisible by 4.

A number is divisible by 4 if the number formed by its digits at tens and ones place is divisible by 4.

V. Divisibility by 8

Let us find if 3364280 is divisible by 8?

Step 1: Separate the number formed by the digits at hundreds, tens and ones place.

3364 / 280

Step 2: Divide 280 by 8.

$$\begin{array}{r|l} & \ \ 35 \\ 8 & 280 \\ & \underline{24} \\ & \ \ 40 \\ & \ \ \underline{40} \\ & \ \ \ \ 0 \end{array}$$

280 is divisible by 4.

Hence, 3364280 is also divisible by 8.

A number is divisible by 8 if the number formed by the digits at hundreds, tens and ones place is divisible by 8.

VI. Numbers with trailing zeroes

Divide 2500 by 4.	Is it divisible?	(Yes/No)
Divide 23900 by 4.	Is it divisible?	(Yes/No)
Divide 34000 by 8.	Is it divisible?	(Yes/No)
Now divide 196000 by 8.	Is it divisible?	(Yes/No)

- **If a number has zeroes in its tens and ones places, it is divisible by 4.**
- **If a number has zeroes in its hundreds, tens and ones places, it is divisible by 8.**

Worksheet 2

1. Look at the following numbers and fill in the blanks.

(a) 435, 6552, 988, 3870, 5211, 9343

The numbers that are divisible by 2 are ________, ________, ________.

(b) 3522, 9765, 1000, 45012, 28775

The numbers that are divisible by 5 are ________, ________, ________.

(c) 7780, 10000, 2567, 57514, 82210

The numbers that are divisible by 10 are ________, ________, ________.

(d) 4924, 63402, 11507, 36572

The numbers that are divisible by 4 are ________, ________, ________.

(e) 789984, 365832, 10098, 395529

The numbers that are divisible by 8 are ________, ________, ________.

2. **Apply the divisibility rule and show that–**

(a) 432566 is divisible by 2

(b) 352115 is divisible by 5

(c) 868060 is divisible by 10

(d) 3496 is divisible by 4

(e) 117904 is divisible by 8

(f) 784300 is divisible by 4

(g) 694000 is divisible by 8

(h) 35088 is divisible by 2

VII. Divisibility by 3

Is 4392126 divisible by 3?

Step 1: Add all the digits of the given number.

4 + 3 + 9 + 2 + 1 + 2 + 6 = 27

Step 2: Divide the sum by 3.

$$\begin{array}{r} 9 \\ 3\overline{)\,27\,} \\ 27 \\ \hline 0 \\ \hline \end{array}$$

27 is divisible by 3

Therefore, 4392126 is also divisible by 3.

A number is divisible by 3 if the sum of its digits is divisible by 3.

VIII. Divisibility by 9

Is 8826921 divisible by 9?

Step 1: Add up all the digits of the given number.

8 + 8 + 2 + 6 + 9 + 2 + 1 = 36

Step 2: Divide the sum by 9.

$$\begin{array}{r} 4 \\ 9\overline{)\,36\,} \\ 36 \\ \hline 0 \\ \hline \end{array}$$

36 is divisible by 9

So, 8826921 is also divisible by 9.

A number is divisible by 9 if the sum of its digits is divisible by 9.

IX. Divisibility by 11

Let us consider a number 13856722. To test whether it is divisible by 11, following steps are taken.

Step 1: Add alternate digits (digits in odd places and digits in even places separately) starting from the ones place.

	O		O		O		O
1	3	8	5	6	7	2	2
E		E		E		E	

Step 2: Sum of the digits at odd places = 2 + 7 + 5 + 3 = 17

Sum of the digits at even places = 2 + 6 + 8 + 1 = 17

Step 3: Difference of the two sums, i.e.

17 – 17 = 0

If the difference between the sum of the digits at even places and sum of the digits at odd places is either 0 or a multiple of 11, the number is divisible by 11.

MORE ON DIVISIBILITY TESTS

I. A number is divisible by 6 if it is divisible by co-prime factors of six.

e.g. 42 is divisible by 2 and 3, therefore, 42 is also divisible by **2 × 3 = 6.**

Similarly,

- A number is divisible by 12 if it is divisible by 4 and 3.
- A number is divisible by 15 if it is divisible by 3 and 5.
- A number is divisible by 24 if it is divisible by 8 and 3.
- A number is divisible by 36 if it is divisible by 9 and 4.

II. If a number is divisible by another number, then it is divisible by each factor of that number.

e.g. 18 is divisible by 6

18 is also divisible by 1, 2, 3 → Factors of 6

III. **If a number is divisible by two co-prime numbers, then it is divisible by their product.**

> Two numbers whose HCF is one are called **Co-prime Numbers.**

e.g. 4 and 3 are co-prime numbers.

24 is divisible by 4

24 is divisible by 3

24 is also divisible by $\underbrace{12}_{4 \times 3}$

IV. **If two given numbers are divisible by a number, then their sum is also divisible by that number.**

e.g. 8 and 12 are divisible by 4

$\underbrace{20}_{8 + 12}$ is also divisible by 4

V. **If two given numbers are divisible by a number, then their difference is also divisible by that number.**

e.g. 15 and 35 are divisible by 5

$\underbrace{20}_{35 - 15}$ is also divisible by 5

Worksheet 3

1. **Look at the following group of numbers and fill in the blanks.**

(a) 389510, 7781450, 4203324, 12342

The numbers divisible by 3 are ____________ and ____________ .

(b) 3437712, 4222910, 6880172, 9811602

The numbers divisible by 9 are ____________ and ____________ .

(c) 362442, 8502153, 774067, 46627207

The numbers divisible by 11 are ____________ and ____________ .

2. **Pick out the numbers from the following that are divisible by 3 but not by 9.**

(a) 38721 (b) 422679 (c) 6110586 (d) 257796

3. **Test the following for the divisibility by 3 and 9.**

(a) 294414 (b) 145404 (c) 99999

4. **Test the divisibility of the following numbers by 11.**

(a) 86611291 (b) 100001 (c) 9427355

(d) 7023643 (e) 58334661 (f) 602111213

5. **Fill in the blanks.**

(a) A number is divisible by 6 if it is divisible by its two co-prime factors ________ and ________ .

(b) 43185 is divisible by 15 as it is divisible by ________ and ________ .

(c) The number 8625 is not divisible by 6 as it is divisible by ________ but not by ________ .

(d) The number 54420 is divisible by 12 as it is divisible by ________ and ________ .

(e) The number 781022 is divisible by 11 as the difference of the sum of the digits at odd places and the sum of the digits at even places is ________ .

6. **Replace $\square$ by a digit so that the number is divisible by 9.**

(a) 384 $\square$ 62 (c) 9080 $\square$

(b) 1 $\square$ 80498 (d) 46 $\square$ 21

7. **Write 'True' or 'False' for the following statements.**

(a) If a number is divisible by 3, it must be divisible by 9. $\square$

(b) If a number is divisible by 18, it must be divisible by 6 and 3. $\square$

(c) If a number is divisible by both 9 and 10, then it must be divisible by 90. $\square$

(d) All numbers which are divisible by 8 are divisible by 4. $\square$

(e) If a number is exactly divisible by two numbers separately then it must be exactly divisible by their sum. $\square$

PRIME FACTORISATION

Let us look at the example given below.

Example 3: Find the prime factorisation of 360 by division method.

Solution:

360 is divisible by the prime number 2

45 is divisible by the prime number 3

We divide by the prime number 5

2	360
2	180
2	90
3	45
3	15
5	5
	1 → We stop

Hence, the prime factorisation of 360 = 2 × 2 × 2 × 3 × 3 × 5

Worksheet 4

1. **Find the prime factorisation of the following.**

   (a) 78 (b) 120 (c) 256 (d) 84 (e) 441

   (f) 240 (g) 2304 (h) 3125 (i) 1260

2. **Write the greatest 4-digit number. Express it as a product of primes.**
3. **Write the smallest 4-digit number and show its prime factorisation.**
4. **Express each of the following numbers as sum of two odd primes.**

   (a) 18 (b) 32 (c) 66 (d) 90

5. **Express the following as sum of three odd primes.**

   (a) 41 (b) 23 (c) 75 (d) 59

HIGHEST COMMON FACTOR (HCF)

HCF of two or more numbers is the **Highest Common Factor** of these numbers.

Let us now find HCF of 27 and 36.

(a) Factor Method

27 My factors are 1, 3, 9, 27

36 My factors are 1, 2, 3, 4, 6, 9, 12, 18, 36

27 Our common factors are 1, 3, 9
Our highest common factor is 9 **36**

So, HCF of 27 and 36 is 9.

Do you know?
HCF is also known as **GCD** which means **Greatest Common Divisor.**

(b) Prime Factorisation Method

My prime factors

27

3	27
3	9
3	3
	1

2	36
2	18
3	9
3	3
	1

My prime factors

36

27 = 3 × 3 × 3 → Prime factors of 27

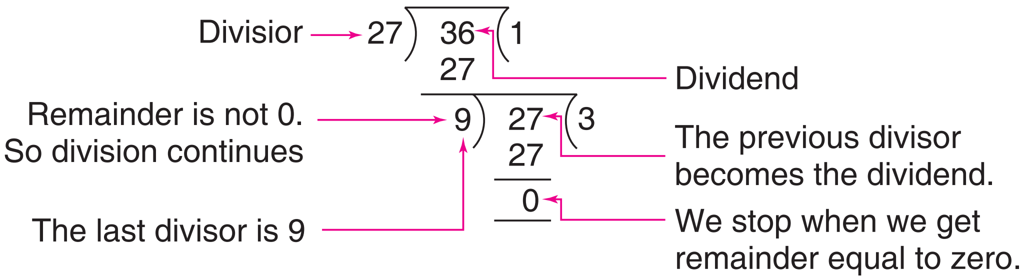
36 = 2 × 2 × 3 × 3 → Prime factors of 36

Common factors of 27 and 36 are 3, 3

Therefore, HCF = 3 × 3 = 9

(c) Continued Division Method

36 is greater than 27, so 36 will be the dividend.

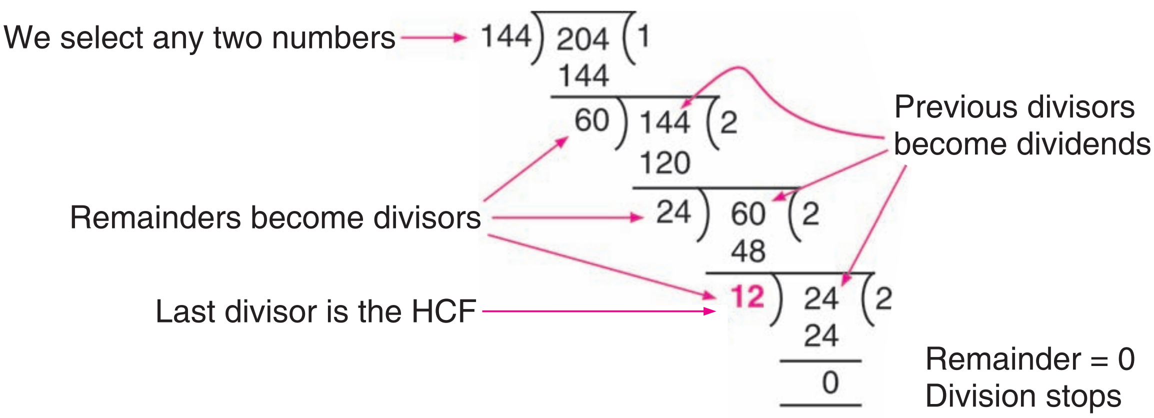


Therefore, HCF is 9.

Let us take some more examples.

Example 4: Find the HCF of 204, 144 and 252.

Solution: Here we have three numbers.



So, HCF of 144 and 204 is 12.

Now, let us find the HCF of 12 and the third number, i.e. 252.

HCF = 12 ⟶ 12) 252 (21
252
0

Remainder = 0
Division stops

So, the HCF of 204, 144 and 252 = 12.

Example 5: Find the greatest number that will divide 140, 170, 155 leaving remainder 5 in each case.

Solution: Here, we have to find a number which exactly divides (140 – 5), (170 – 5), (155 – 5)

The required number is the HCF of 135, 165 and 150.

First take any two numbers, say 135 and 165.

135) 165 (1
135
30) 135 (4
120
HCF ⟶ 15) 30 (2
30
0

HCF of 165 and 135 is 15.

Now, we find the HCF of 15 and 150

15) 150 (10
150
0

The required number is 15.

Example 6: The floor of a room is 6 m 75 cm long and 5 m wide. It is to be paved with square tiles. Find the largest size of tile needed.

Solution: In order to find the largest size of tile needed, we find the number that divides 675 and 500 exactly.

500) 675 (1
500
175) 500 (2
350
150) 175 (1
150
HCF ⟶ **25**) 150 (6
150
0

6 m 75 cm = 675 cm
5 m = 500 cm

The largest size of tile needed is 25 cm.

Worksheet 5

1. **Find the HCF of the following numbers by factor method.**

   (a) 7, 18 (b) 12, 30, 54 (c) 70, 14, 35

2. **Find the HCF of the following numbers by prime factorisation method.**

   (a) 76, 28 (b) 24, 16, 36 (c) 38, 64, 82

3. **Find the HCF of the following numbers by continued division method.**

   (a) 345, 506 (b) 144, 384, 120

   (c) 287, 533 (d) 208, 494, 949

   (e) 1212, 6868, 1111 (f) 1794, 2346, 4761

   (g) 70, 105, 175 (h) 270, 450, 315

4. **What is the HCF of–**

   (a) two consecutive natural numbers.

   (b) two consecutive even numbers.

   (c) two consecutive odd numbers.

   (d) any two prime numbers.

5. **Find the greatest number which divides 203 and 434 leaving remainder 5 in each case.**

6. **Find the greatest number which will divide 625 and 1433 leaving remainders 5 and 3 respectively.**

7. **The length, breadth and height of a room are 8.25 m, 6.75 m and 4.50 m respectively. Determine the longest tape which can measure the three dimensions of the room exactly.**

8. **There are 312 mango bites, 260 eclairs and 156 coffee bites in a box. These are to be put in packets so that each packet contains the same number of toffees. Find the maximum number of toffees in each packet.**

LEAST COMMON MULTIPLE (LCM)

LCM of two or more numbers is the **Least Common Multiple** of these numbers.

Let us find the LCM of 3, 6 and 9.

(a) By Listing Multiples

3 — My multiples are 3, 6, 9, 12, 15, 18, 21 ......

6 — My multiples are 6, 12, 18, 24, 30 ......

9 — My multiples are 9, 18, 27, 36, 45 ......

The least common multiple of these three numbers is 18.

So, LCM of 3, 6, 9 is 18.

(b) Prime Factorisation Method

Let us find the LCM of 24, 15 and 45.

Divide by prime number till you get 1 as quotient.

$$\begin{array}{r|r} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

We have, $24 = 2 \times 2 \times 2 \times 3$

$15 = 3 \times 5$

$45 = 3 \times 3 \times 5$

Therefore, LCM $= \underbrace{2 \times 2 \times 2} \times \underbrace{3 \times 3} \times 5$

$= 8 \times 9 \times 5 = 360$

2 occurs maxiumum three times, 3 occurs two times and occurs one time only.

(c) Common Division Method

Let us find the LCM of 30, 45, 60.

2	30, 45, 60
2	15, 45, 30
3	15, 45, 15
3	5, 15, 5
5	5, 5, 5
	1, 1, 1

Divide the numbers by the common prime factor of one or more numbers.

Repeat the same process of division.

Write numbers in a line separated by commas.

45 is not divisible by 2. Write it as it is.

Stop when you get all quotients equal to one.

$$\text{LCM} = 2 \times 2 \times 3 \times 3 \times 5$$
$$= 180$$

Let us solve some examples.

Example 7: Find the smallest number which when divided by 25, 40, 60 leaves remainder 7 in each case.

Solution: The required number will be 7 added to the least common multiple (LCM) of these numbers.

Let us first find the LCM.

2	25, 40, 60
2	25, 20, 30
2	25, 10, 15
3	25, 5, 15
5	25, 5, 5
5	5, 1, 1
	1, 1, 1

$$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 5 \times 5$$
$$= 600$$

Therefore, the required number = 600 + 7 = 607.

Let us check.

25) 607 (24
50
107
100
7

40) 607 (15
40
207
200
7

60) 607 (10
60
7

See in all cases the remainder is 7.

Example 8: In a morning walk, three boys step off together. Their steps measure 80 cm, 85 cm and 90 cm respectively. What minimum distance should each walk so that all can cover the distance in complete steps?

Solution: The minimum distance needed will be the Least Common Multiple (LCM) of 80, 85, 90.

2	80, 85, 90
2	40, 85, 45
2	20, 85, 45
2	10, 85, 45
3	5, 85, 45
3	5, 85, 15
5	5, 85, 5
17	1, 17, 1
	1, 1, 1

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 17$$

$$= 12240$$

Required distance = 12240 cm

or 122 m 40 cm

Worksheet 6

1. **Find LCM of the following numbers by listing their multiples.**

   (a) 12, 9 (b) 4, 5, 2 (c) 25, 15

2. **Find the LCM by prime factorisation method.**

   (a) 10, 15, 6 (b) 16, 12, 18 (c) 25, 30, 40

3. **Find the LCM by common division method.**

   (a) 12, 15, 45 (b) 24, 90, 48

   (c) 30, 24, 36, 16 (d) 16, 48, 64

   (e) 35, 49, 91 (f) 20, 25, 30

   (g) 12, 16, 24, 36 (h) 40, 48, 45

4. **Find the least number which when divided by 40, 50 and 60 leaves remainder 5 in each case.**

5. **Three Haryana Roadways buses stop after 50, 100 and 125 km respectively. If they leave together, then after how many kilometres will they stop together?**

6. **Four bells toll at intervals of 8, 9, 12 and 15 minutes respectively. If they toll together at 3 p.m., when will they toll together next?**

PROPERTIES OF HCF AND LCM

1. **HCF of given numbers is not greater than any of the numbers.**

   e.g. HCF of 5 and 15 = 5

   HCF of 12 and 18 = 6

2. **LCM of given numbers is not smaller than any of the numbers.**

   e.g. LCM of 5, 15 = 15

   LCM of 12, 18 = 36

3. **HCF of given numbers is a factor of their LCM.**

   e.g. HCF of 16, 12 = 4

   LCM of 16, 12 = 48

   HCF 4 is a factor of LCM 48

4. **LCM of given numbers is a multiple of their HCF.**

   e.g. HCF of 16, 12 = 4

   LCM of 16, 12 = 48

   LCM 48 is a multiple of HCF 4

5. **If HCF of two numbers is one of the number then LCM is the greater number.**

   e.g. HCF of 5 and 15 = 5

   LCM = 15 (greater number)

6. **HCF of co-prime numbers is 1.**

   5 and 9 are co-prime

   HCF = 1

7. **LCM of co-prime numbers is the product of the numbers.**

   LCM of 5 and 9 = 45

8. **Product of HCF and LCM of two numbers is equal to the product of the numbers.**

   HCF of 9 and 12 = 3

   LCM of 9 and 12 = 36

   Product of 9 and 12 = 108

   Product of HCF and LCM = $3 \times 36 = 108$

Let us study some examples.

Example 9: Find HCF and LCM of 25, 65.

Solution: Here, we find only HCF of 25 and 65.

HCF of 25, 65 ⟶

$$
\begin{array}{l}
25)\ 65\ (2 \\
\quad\ \ 50 \\
\quad 15)\ 25\ (1 \\
\qquad\quad 15 \\
\qquad 10)\ 15\ (1 \\
\qquad\qquad\ 10 \\
\text{HCF} \longrightarrow \quad 5)\ 10\ (2 \\
\qquad\qquad\qquad 10 \\
\qquad\qquad\qquad\ 0
\end{array}
$$

HCF = 5

LCM will be found by using the property.

Product of numbers = Product of HCF and LCM.

$$25 \times 65 = 5 \times \text{LCM}$$

$$\text{LCM} = \frac{\cancel{25}^{5} \times 65}{\cancel{5}_{1}} = 325$$

Example 10: HCF of two numbers is 16 and their product is 6400. Find their LCM.

Solution: We have,

$$\text{HCF} \times \text{LCM} = \text{Product of numbers}$$

$$16 \times \text{LCM} = 6400$$

$$\text{LCM} = \frac{\cancel{6400}^{400}}{\cancel{16}_{1}} = 400$$

$$\text{LCM} = 400$$

Worksheet 7

1. **For each of the following pairs of numbers, verify that product of numbers is equal to the product of their HCF and LCM.**

   (a) 10, 15 (b) 35, 40 (c) 32, 48

2. **Find HCF and LCM by using the property in Question no. 1.**

   (a) 27, 90 (b) 145, 232 (c) 117, 221

3. **Can two numbers have 16 as HCF and 380 as LCM? Give reasons.**

4. **The HCF of two numbers is 16 and product of numbers is 3072. Find their LCM.**

5. **The LCM and HCF of two numbers are 180 and 6 respectively. If one of the number is 30, find the other.**

6. **LCM of two numbers 160 and 352 is 1760. Find their HCF.**

7. **Write 'True' or 'False' for the following statements.**

(a) LCM of two numbers is a factor of their HCF.

(b) Product of three numbers is equal to the product of their HCF and LCM.

(c) HCF of given numbers is always a factor of their LCM.

(d) LCM of given numbers cannot be smaller than the numbers.

(e) LCM of co-prime numbers is equal to their product.

VALUE BASED QUESTIONS

1. **The schools nowadays have a council which consists of student representatives. This council helps the school in organising various events. Rohan has also been selected in his school council this year.**

The school organised a picnic in which 108, 162 and 270 students of Classes-VI, VII and VIII respectively were going. Rohan's teacher asked him to help the transport incharge.

(a) Find out the number of buses required, if each bus had to carry maximum but equal number of students from each class.

(b) As a member of school council Rohan was made a part of decision making. What other values does the student council develop in a child?

(c) Suggest one way by which you can help your school if you are selected as a council member.

2. **You know 5 June is celebrated every year as World Environment Day. As a part of its celebration, Vrinda and her two friends decided to have a cycling race to promote environmental friendly transport. They started at 12 noon and took 3 minutes 20 seconds, 3 minutes 40 seconds and 4 minutes respectively to cycle on a circular track.**

(a) If Vrinda and her friends started together at 12 noon, then when will they meet next?

(b) Suggest the ways by which you can save the environment.

BRAIN TEASERS

1. A. Tick (✓) the correct answer.

(a) Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. After how many minutes will they toll together again?

(i) 5 minutes (ii) 6 minutes

(iii) 4 minutes (iv) 2 minutes

(b) The HCF of two numbers is 11 and their LCM is 7700. If one of the numbers is 275, then the other number is–

(i) 279 (ii) 283 (iii) 308 (iv) 318

(c) The greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm is–

(i) 35 cm (ii) 25 cm (iii) 15 cm (iv) 42 cm

(d) 252 can be expressed as a product of primes as–

(i) $2 \times 2 \times 3 \times 3 \times 7$ (ii) $2 \times 2 \times 2 \times 3 \times 7$

(iii) $3 \times 3 \times 3 \times 3 \times 7$ (iv) $2 \times 3 \times 3 \times 3 \times 7$

(e) Which of the following is a factor of every natural number?

(i) 1 (ii) 0 (iii) – 1 (iv) any number

B. Answer the following questions.

(a) Find the highest common factor of 36 and 84.

(b) How many factors does 36 have?

(c) Express 132 as the sum of two odd primes.

(d) What should be added to 4057 to make it divisible by 9?

(e) Find the HCF of 95, 105 and 115 by continued division.

2. Are 32 and 34 co-prime numbers? Why?

3. Write any four twin primes between 50 and 110.

4. Express the greatest 3-digit number as a product of primes.

5. **Express the smallest 5-digit number as a product of primes.**

6. **State which of the following numbers are divisible by both 3 and 9?**

   (a) 235674 (b) 78015

7. **Test which of the following numbers are divisible by 11.**

   (a) 147246 (b) 2352825

8. **What least number should be subtracted from the following numbers to make them divisible by 3?**

   (a) 2825 (b) 856291

9. **What least number should be added to the following numbers to make them divisible by 9?**

   (a) 42724 (b) 39065

10. **Replace the blank in 625 $\boxed{}$ with the least number, so that the number is divisible by 11.**

11. **Write any two numbers which are–**

    (a) divisible by 3 but not 9.

    (b) divisible by 5 but not 10.

    (c) divisible by both 4 and 8.

    (d) divisible by 2, 4 and 8.

12. **Find the HCF of 1624, 522 and 1276.**

13. **Find the LCM of 198, 135, 108 and 54.**

14. **The HCF and LCM of two numbers are 13 and 1989 respectively. If one number is 117, find the other.**

15. **Can two numbers have 15 as HCF and 350 as LCM? Why?**

HOTS

Find the greatest number of four digits which is divisible by 15, 20 and 25.

ENRICHMENT QUESTION

To find the factors of a number, you have to find all the pairs of numbers that multiply together to give that number.

The factors of 48 are:

1 and 48 2 and 24 3 and 16 4 and 12 6 and 8

If we leave out the number we started with, 48, and add all the other factors, we get 76:

$$1 + 2 + 3 + 4 + 6 + 8 + 12 + 16 + 24 = 76$$

So ... 48 is called an **abundant number** because it is less than the sum of its factors (without itself). (48 is less than 76.)

A number less than the sum of its factors except itself is called an **abundant number.**

See if you can find some more abundant numbers!

YOU MUST KNOW

1. Two prime numbers whose difference is 2 are called twin prime numbers.
2. Two numbers are said to be co-prime when they have only 1 as common factor.
3. Every number has infinite number of multiples and finite number of factors.
4. A number is divisible by another number if it is divisible by its co-prime factors.
5. If a number is divisible by another number, then it is divisible by each factor of that number.
6. If a number is divisible by two co-prime numbers, then it is divisible by their product.
7. If two given number are divisible by a number, then their sum is also divisible by that number.
8. If two given number are divisible by a number, then their difference is also divisible by that number.
9. Prime factorisation of a number is the factorisation in which every factor is a prime number.
10. HCF is also known as the Greatest Common Divisior (GCD).
11. Product of HCF and LCM of two numbers is equal to the product of the numbers.
12. HCF of given numbers is not greater than any of the numbers.
13. LCM of given numbers is not smaller than any of the numbers.

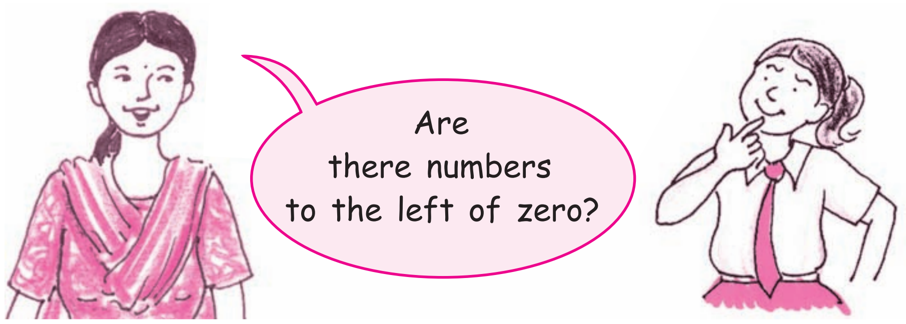
CHAPTER 3 INTEGERS

INTRODUCTION

NEED FOR INTEGERS

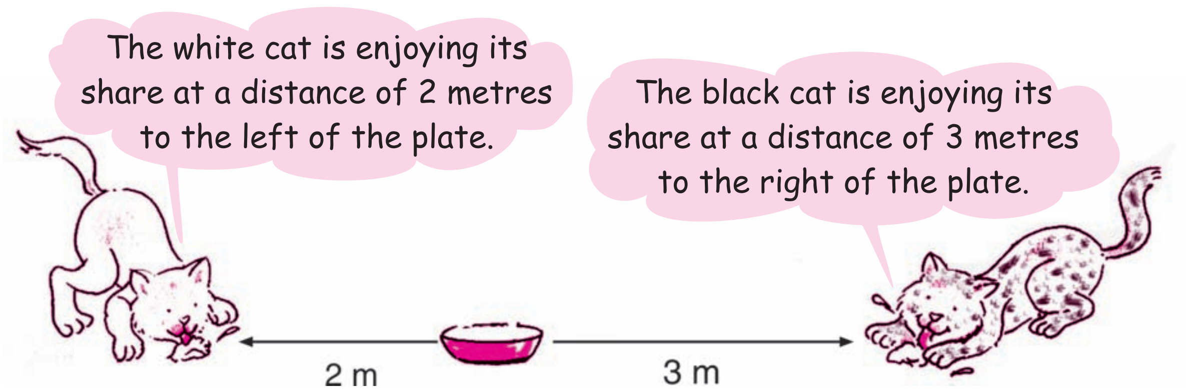
Observe the number line drawn below.

On the number line, 0 (zero) is the starting point (called the **origin**) and all the natural numbers are to the right of 0.



Now let us consider some situations.

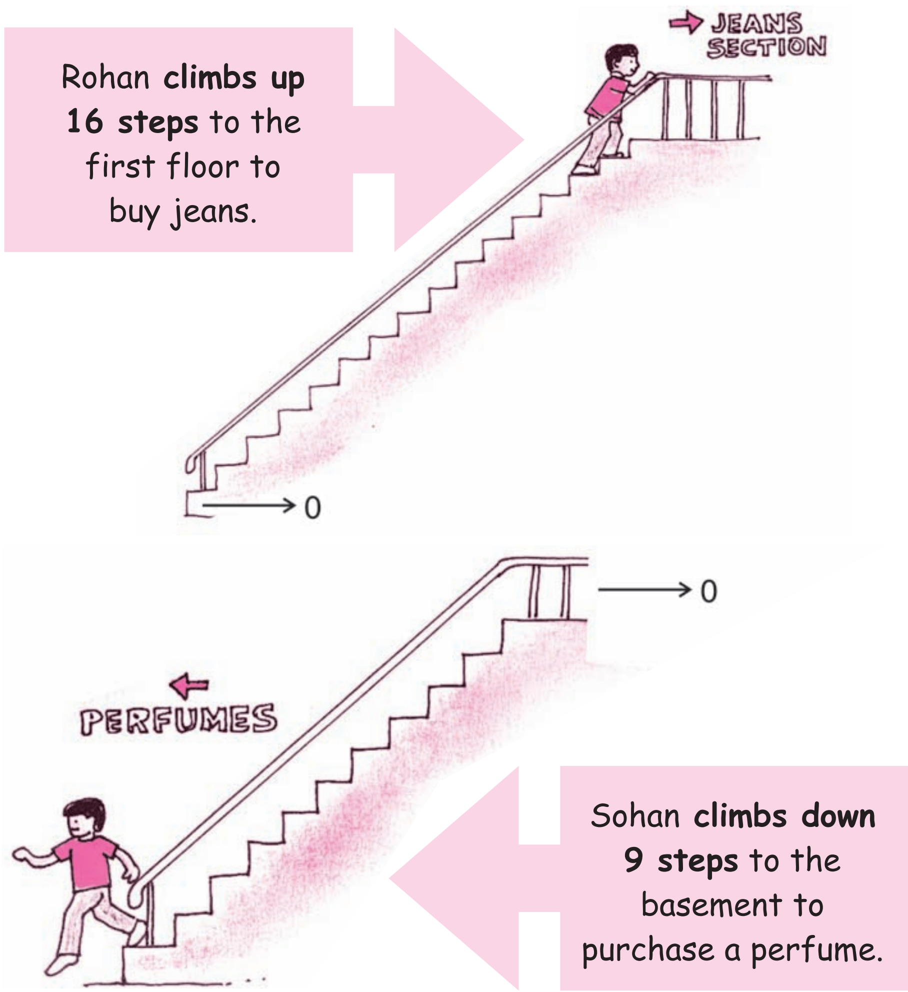
Situation 1: See! these two cats have pounced on a piece of cake that was on the plate.



Let us take the plate as the starting point 0. We have two numbers on the opposite sides of 0.

3 m to the right of 0 and 2 m to the left of 0.

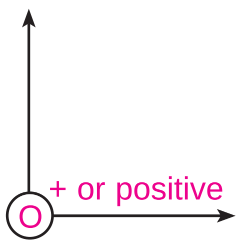
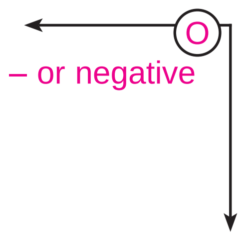
Situation 2: See! Rohan and Sohan are going to a shop to make some purchases.



Now, let us take the ground level as origin 0. Here, we have 2 numbers on the opposite sides of 0.

16 steps **above** 0 and 9 steps **below** 0.

To distinguish numbers on the opposite sides of zero, that is **right** and **left** or **above** and **below**, we use opposite signs, i.e. positive (**+**) and negative (**–**).

Positive (+) means to the right of or above the origin.	Negative (–) means to the left of or below the origin.
	

In the above two situations,

3 m to the right of 0 is represented as	**+ 3**
2 m to the left of 0 is represented as	**– 2**
climbing up 16 steps is represented as	**+ 16**
climbing down 9 steps is represented as	**– 9**

Similarly,

A profit of ₹ 200 is	**+ 200**
8°C below the freezing point is	**– 8**
Depositing ₹ 500 in a bank is	**+ 500**

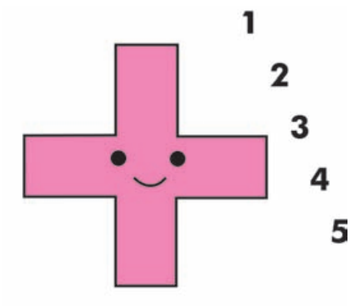
Numbers with positive (**+**) sign are called **positive numbers**.

Numbers with negative (**–**) sign are called **negative numbers**.

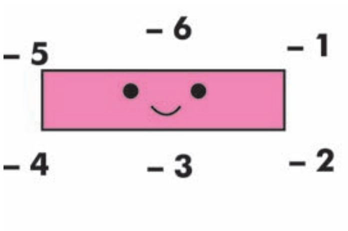
Remember

Positive numbers, negative numbers along with zero are called **Integers.**

Positive numbers 1, 2, 3, 4, ... are called **positive integers.**



Zero is neither negative nor positive.



Negative numbers –1, –2, –3, –4, ... are called **negative integers.**

Negative integers – 1, – 2, – 3, ... are read as minus one, minus two, minus three, etc.

OPPOSITES

- Opposite of the **PROFIT** of ₹ 20 is **LOSS** of ₹ 20.
- Opposite of 5°C **ABOVE** freezing point is 5°C **BELOW** freezing point.
- Opposite of – 3 is + 3.

Worksheet 1

1. **Indicate the following by using integers.**

   (a) Earning ₹ 500

   (b) Loss of ₹ 90

   (c) Climbing up 10 steps

   (d) Withdrawal of ₹ 500 from a bank

   (e) 5 m above sea level

   (f) 3 km towards north

   (g) 10°C below zero

   (h) An increase of 25 marks

2. **Write the opposites of–**

   (a) Depositing ₹ 1,000 in a bank account.

   (b) Decrease of 5 marks.

   (c) Earning ₹ 200.

   (d) Going 2 km towards east.

   (e) Two steps to the left of zero on a number line.

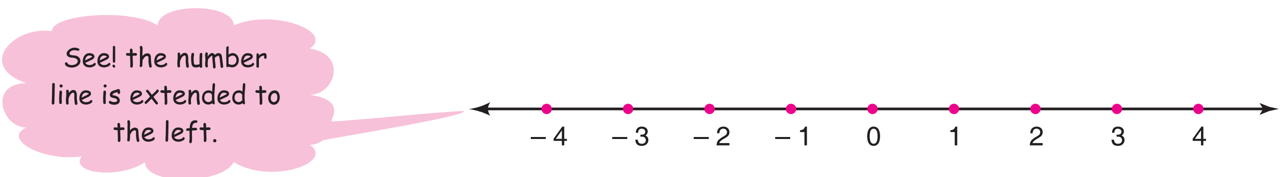
   (f) Losing weight of 7 kg.

3. **Encircle the negative integers from the following numbers.**

   – 59, 6, 0, – 1, – 4, 45, – 62, 107

REPRESENTATION OF INTEGERS ON A NUMBER LINE

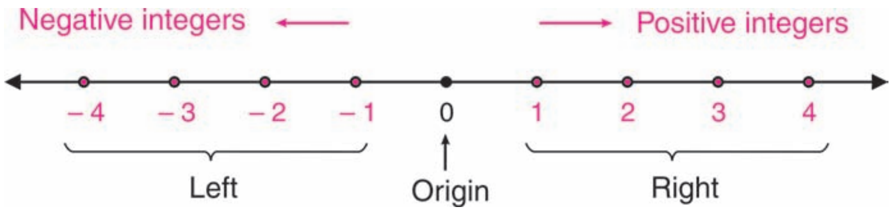
We know that negative integers are opposite of positive integers. So let us mark the negative integers on the left of zero on the number line.

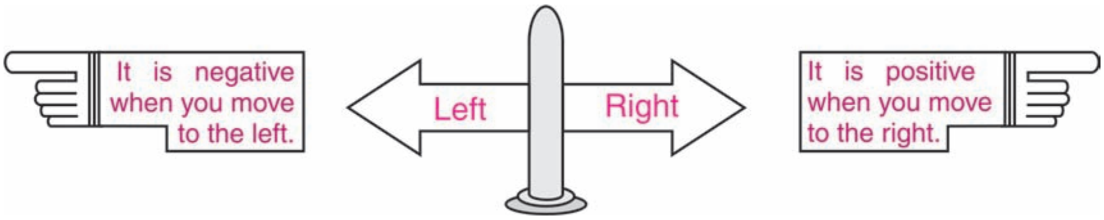


Note:

- The opposite integers (e.g. – 2 and + 2) are at the same distance from zero.
- The distance between consecutive integers is same everywhere.

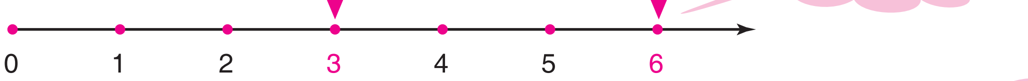
So, now we have the number line...



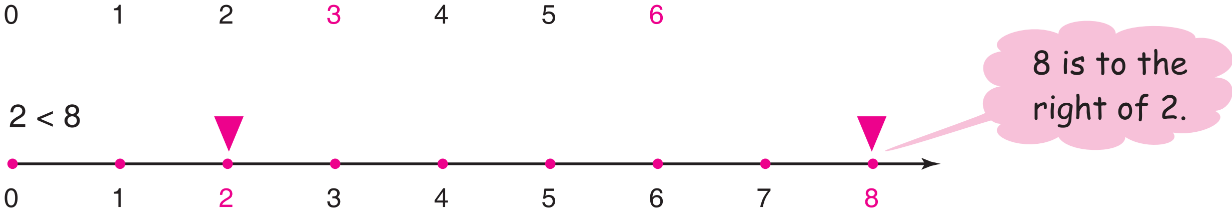


ORDERING OF INTEGERS

We have,
6 > 3



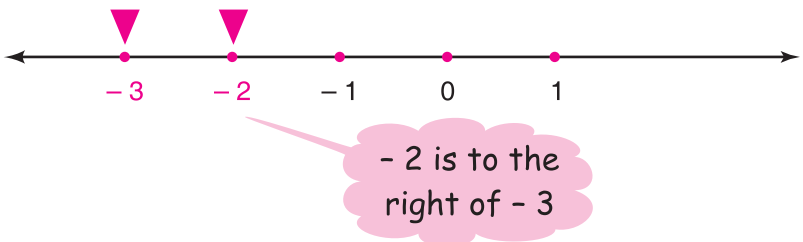
2 < 8



Remember

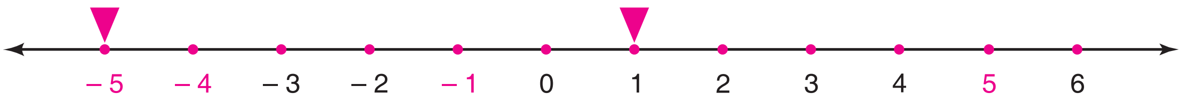
A number to the right of a given number is greater than the given number.

Now, let us compare – 2 and – 3.



So, – 2 is greater than – 3
or – 2 > – 3

Compare +1 and – 5



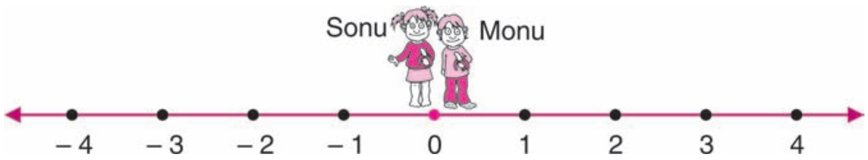
1 is to the right of – 5

So, 1 is greater than – 5
or 1 > – 5.

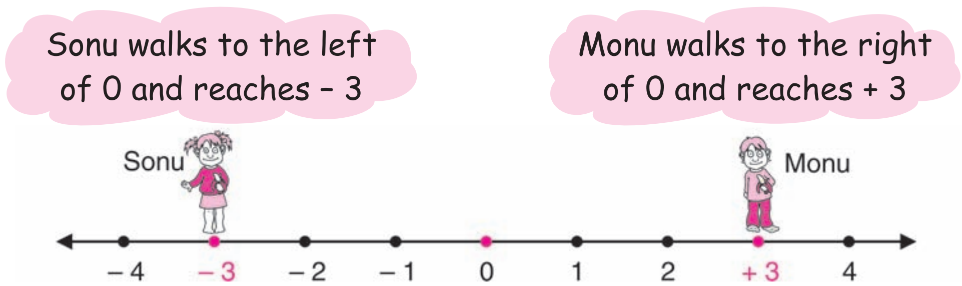
Note:

- Every positive integer is greater than any negative integer.
- Zero is less than every positive integer.
- Zero is greater than every negative integer.
- – 1 is the greatest negative integer.
- We cannot find the greatest positive integer or the smallest negative integer.

ABSOLUTE VALUE OF INTEGERS



See! Sonu and Monu are standing at a point zero (0). After two minutes, see their position.



Remember

Absolute value of an integer is its numerical value, without taking the sign into account.

Here the distance walked by both of them is same (3 units) without taking into account the direction (sign). So, we can say that the absolute value of 3 and – 3 is **3**.

Let us find the absolute value of some integers.

The absolute value of an integer is greater than or equal to the integer.

Integer	Absolute Value
+ 1	1
– 1	1
– 7	7
21	21
0	0

The absolute value of any non-zero integer is positive.

The symbol used to write absolute value is **two vertical lines (| |)**, one on either side of the integer.

Thus, the absolute value of – 7 is written as | – 7 | = 7

Worksheet 2

1. **Do as directed.**

   (a) Mark any point as origin on the given number line.

   (b) Write integers on either side of the origin with proper signs.

2. **Encircle the number which is to the right of the other on number line in each of the following pairs.**

   (a) 3, – 1 (b) 0, – 8

   (c) – 6, – 4 (d) 14, – 7

   (e) – 9, – 8 (f) 4, 7

3. **Write all the integers between–**

   (a) – 5 and 0 (b) – 4 and 3

   (c) – 11 and 1 (d) – 6 and – 1

4. **Compare the numbers and insert an appropriate symbol (>, <, =) in the given space.**

   (a) 3 ◯ – 3 (b) – 1 ◯ 0

   (c) – 101 ◯ – 104 (d) – 82 ◯ – 28

   (e) – 4 ◯ – 14 (f) 16 ◯ – 16

   (g) – 97 ◯ – 98 (h) – 197 ◯ – 96

   (i) 0 ◯ – 7 (j) – 1 ◯ 1

5. **Fill in the following table with the absolute values.**

Integer	Absolute value
17	
– 18	
0	
– 43	
21	
– 105	
– 61	
1283	

6. **Write the following in ascending order.**

   (a) 4, – 5, 16, – 11, – 21, 50

   (b) 0, – 1, 7, – 16, – 12, – 30

7. **Write the following in descending order.**

   (a) – 171, 26, – 43, 103, – 105, 77

   (b) 9, – 8, 0, – 75, – 79, 93

8. **Write 'True' or 'False' for the following statements.**

   (a) Every integer is either positive or negative.

   (b) Zero is greater than every negative integer.

   (c) An integer to the left of another integer is always smaller.

   (d) We can find the smallest integer.

   (e) Absolute value of a given integer is always greater than the integer.

   (f) All natural numbers are positive integers.

   (g) All whole numbers are integers.

   (h) Absolute value of 3 is – 3.

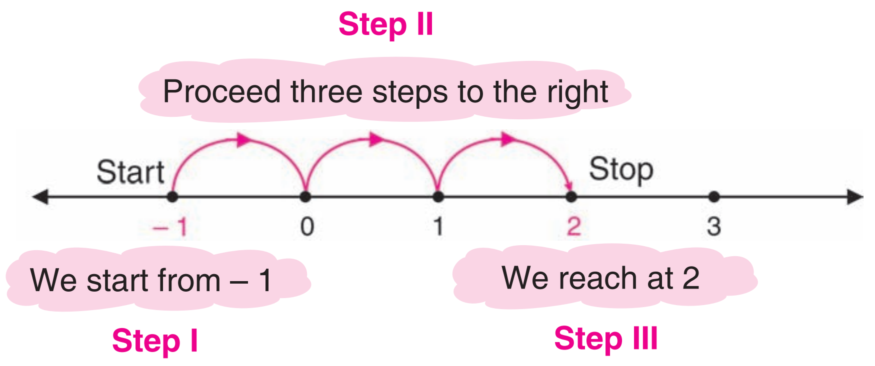
OPERATIONS ON INTEGERS

A. ADDITION OF INTEGERS

Let us find the position of following numbers on number line.

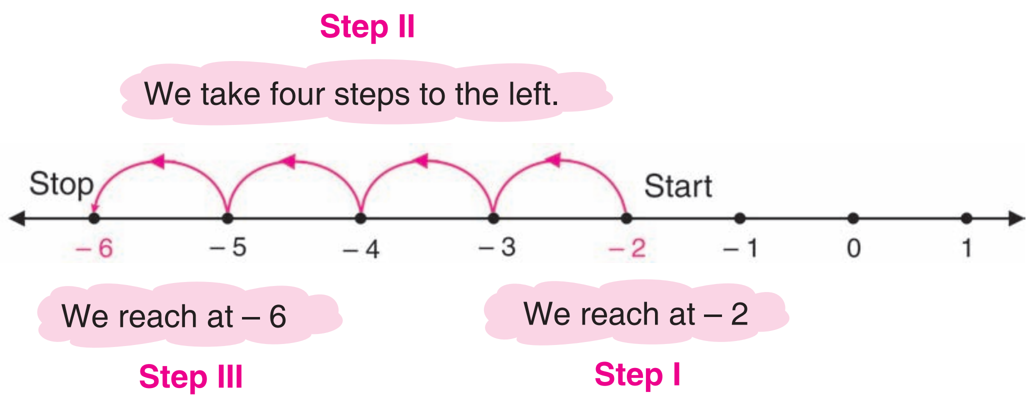
(a) 3 more than – 1 (b) 4 less than – 2

(a) 3 more than – 1



So, the number 3 more than – 1 is 2.

(b) 4 less than – 2



So, the number 4 less than – 2 is – 6.

To find a number more than a given number, we proceed to the right and to find a number less than a given number, we go to the left.

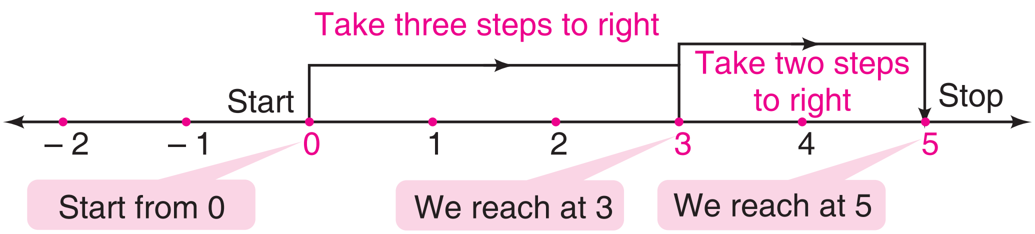
Now, let us perform the operation of addition on the number line.

(i) Addition of two positive integers

Add (+ 3) and (+ 2) on a number line.

Remember

+2 means 2 steps towards right.



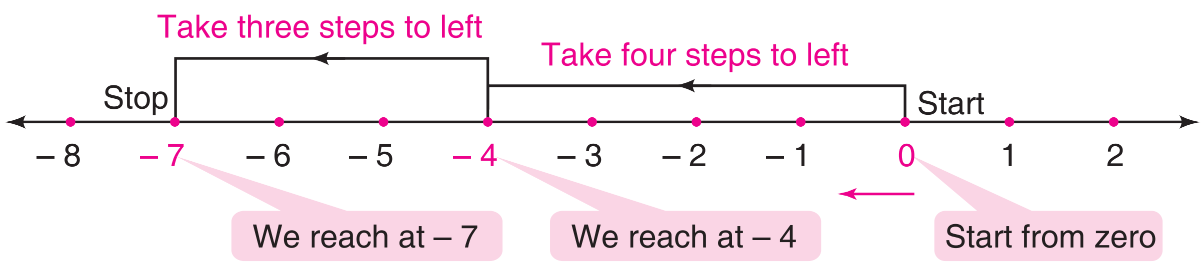
So, (+ 3) + (+ 2) = (+ 5).

Remember

– 3 means 3 steps to left.

(ii) Addition of two negative integers

Add (– 4) + (– 3)



So, (– 4) + (– 3) = (– 7)

Let us do these sums without the help of number line.

Example 1: Add (+ 3) and (+ 2)

Solution: $\left.\begin{array}{l} |+3| = 3 \\ |+2| = 2 \end{array}\right\}$ We take the absolute values of integers.

3 + 2 = 5 — We add the absolute values.

(+ 3) + (+ 2) = + 5 — We prefix the sign of addends in their sum.

Example 2: Add (– 4) and (– 3)

Solution: (– 4) + (–3)

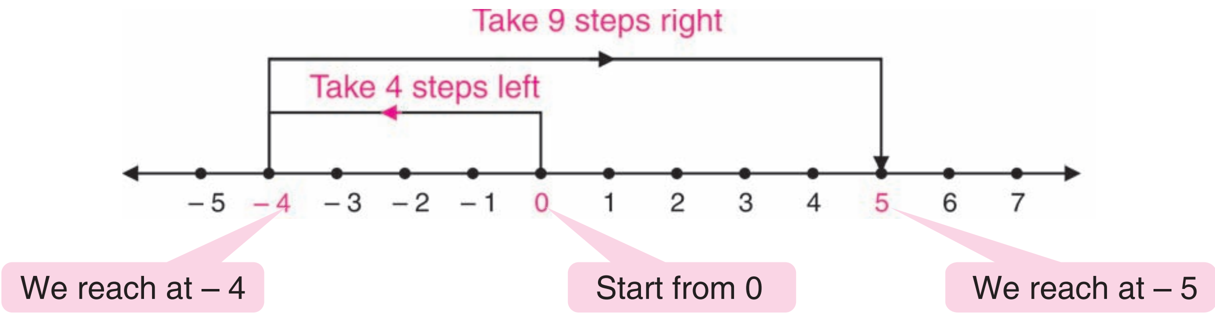
$\left.\begin{array}{l} |-4| = 4 \\ |-3| = 3 \end{array}\right\}$ We take absolute values.

$\left.\begin{array}{l} (-4) + (-3) = -(4 + 3) \\ \qquad\qquad\quad\;\; = -7 \end{array}\right\}$ We add the absolute values and prefix the sign of addends.

> **To add two positive integers or two negative integers, add their absolute values and prefix the sign of addends to the sum.**

(iii) Adding one positive and one negative integer

Let us add – 4 and + 9.



So, (– 4) + (+ 9) = (+ 5)

We can also do this sum without the help of number line.

$|-4| = 4$
$|9| = 9$ — We take the absolute values.

$= +(9 - 4)$ — We find the difference of absolute values.

$= +5$ — We prefix the sign of the integer whose absolute value is greater.

> **If integers have opposite signs, we find the difference of their absolute values and prefix the sign of the integer whose absolute value is greater.**

Worksheet 3

1. **Use the number line and write the number which is:**

   (a) 3 more than 4 (b) 5 less than 1

   (c) 7 more than – 8 (d) 2 less than 2

   (e) 5 more than 6 (f) 7 less than 0

2. **Find the sum on a number line.**

   (a) 8 + (– 3) (b) – 7 + 2

   (c) (– 5) + (– 4) (d) (– 2) + 1 + (– 2)

   (e) 7 + (– 4) + (– 3) (f) (– 2) + (– 3) + (– 4)

3. **Add the following:**

(a)	67, – 49	(b)	– 452, 138
(c)	– 95, – 35	(d)	6951, – 6952
(e)	1001, – 101	(f)	– 381, – 619
(g)	– 419, 386, 419	(h)	– 19, 158, – 103
(i)	– 9005, 360	(j)	– 65, – 35, 100

PROPERTIES OF ADDITION

Property-1: **The sum of any two integers is also an integer.**

Let us add + 5 and – 9

$$+5 + (-9) = -4 \longrightarrow \boxed{(-4) \text{ is also an integer.}}$$

Integers

Property-2: **The sum remains the same even if we change the order of the addends.**

Consider the sum of – 6 and + 11

(– 6) + (+ 11) = + 5

We also have 11 + (– 6) = + 5

Sum is same

The order of addends is changed.

Property-3: **Sum of three integers remains the same even after changing the grouping of the addends.**

Now, add 3, – 5, 9

[3 + (– 5)] + 9]	First we add 3 and (– 5)
= (– 2) + 9	We add the sum to 9
= 7	

Now, let us change the groupings.

3 + [(– 5) + 9]	Grouping is changed
= 3 + 4	We add the sum to 3
= 7	

See! Sum remains the same.

Property-4: **When zero is added to any integer, the sum is the integer itself.**

We have, $3 + 0 = 0 + 3 = 3$

$$-11 + 0 = 0 + (-11) = -11$$

Note: **Zero** is called the identity element for addition.

Property-5: **When one is added to any integer, we get its successor.**

Let us see what will happen when we add one to any integer.

$10 + 1 = +11$	$+11$ is the successor of $+10$
$-7 + 1 = -6$	-6 is the successor of -7

Property-6: **Every integer has an additive inverse such that their sum (integer and additive inverse) is equal to zero.**

Consider the following sums.

$5 + (-5) = 0$	-5 is the opposite of 5
$-8 + 8 = 0$	8 is the opposite of -8

The opposite of an integer is also called the **negative** or **additive inverse** of the integer.

Worksheet 4

1. **Find the sum in two different ways.**

   (a) $-32, 50$ (b) $-81, -79$

   (c) $64, -100$ (d) $13, -78, 15$

2. **Write the additive inverse of the following:**

   (a) 31 (b) -7

   (c) 21 (d) -501

   (e) 0 (f) -34

3. **Find the sum using the properties of addition.**

   (a) $200 + (-105) + (-36)$

   (b) $(-45) + 100 + (-55)$

   (c) $(-825) + 725 + 100 + (-100)$

   (d) $927 + (-517) + (-518)$

(e) (– 215) + (– 215) + 860 + (– 215) + (– 215) + 1

(f) 305 + (– 5) + (– 2) + 2 + (– 200)

(g) 637 + 350 + (– 237) + (– 900)

(h) (– 99) + 7 + (– 101) + 93

4. Fill in the following blanks.

(a) The negative of – 3 is $\boxed{}$

(b) (– 8) + $\boxed{}$ = (– 8)

(c) 11 + (– 16) = $\boxed{}$ + 11

(d) [(– 3) + 5] + 6 = (– 3) + [$\boxed{}$ + $\boxed{}$]

(e) The identity element of addition is $\boxed{}$

(f) (– 51) + 51 = $\boxed{}$

5. Write 'True' or 'False' for the following statements.

(a) 3 + (– 5) is not an integer. $\boxed{}$

(b) Sum of two negative integers is also a negative integer. $\boxed{}$

(c) Negative of – 15 does not exist. $\boxed{}$

(d) [(– 3) + 8] + (– 4) = [8 + (– 3)] + (– 4) $\boxed{}$

(e) 91 + (– 41) = (– 91) + 41 $\boxed{}$

(f) – 46 + 0 = 0 $\boxed{}$

(g) Sum of a positive integer and a negative integer is always negative. $\boxed{}$

(h) | – 9 – 5 | = | – 9 | – | 5 | $\boxed{}$

B. SUBTRACTION OF INTEGERS

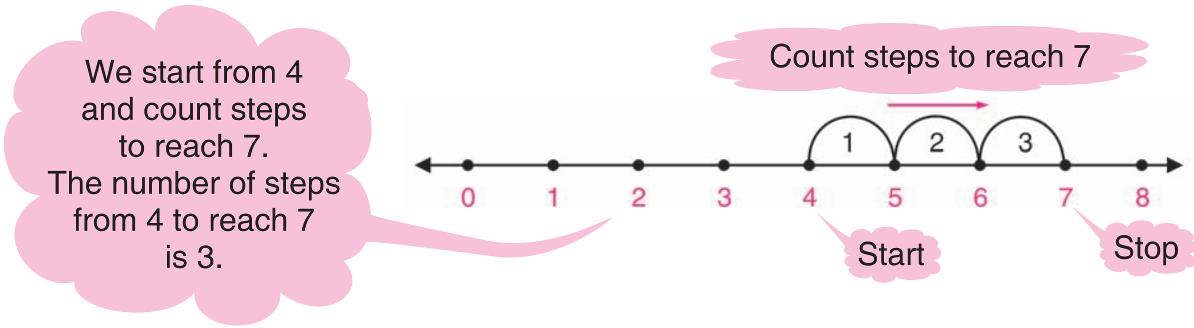
Remember

Subtraction is the inverse of addition.

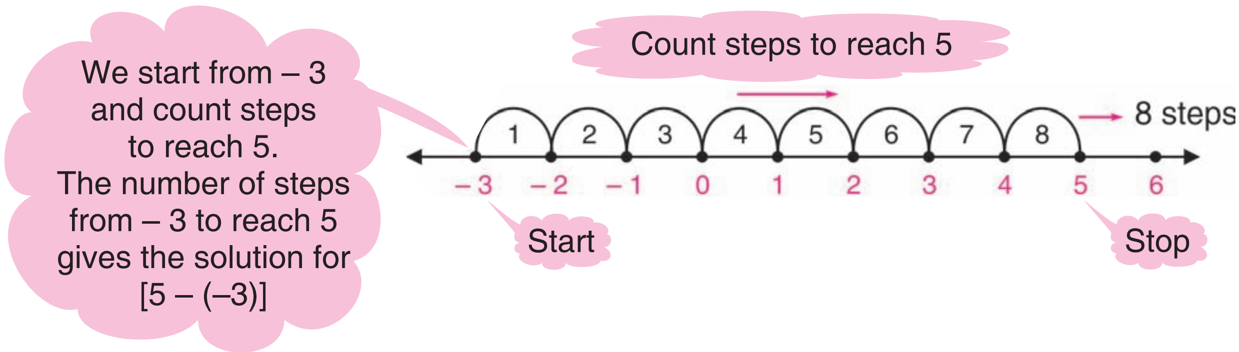
Subtract 4 from 7

If 7 – 4 = 3, then 4 + 3 = 7

Using a number line



Suppose, we want to subtract – 3 from 5, i.e. 5 – (– 3).



Remember

Negative of a negative integer is the corresponding positive integer.

e.g. 5 – (– 3) = 5 + 3 = 8

So, 5 – (– 3) = 8 Adding negative of integer

Let us do more examples.

Example 3: Subtract 2 from – 6

Solution:

– 6 – (+ 2) 2 means + 2

= – 6 + (– 2) negative of + 2

adding – 6 and – 2

= – 8

Example 4: Subtract – 3 from – 10

Solution: We have, – 10 – (– 3)

negative of – 3

= – 10 + 3

adding negative of + 3

= – 7

To subtract two integers, we add the negative of the subtrahend to the minuend.

PROPERTIES OF SUBTRACTION

Property-1: The difference of any two integers is also an integer.

e.g. $3 - (+5) = -2$ $\quad$ -2 is an integer.

Property-2: Every integer has its predecessor.

e.g. the predecessor of -5 is $(-5) - 1 = -6$

Property-3: Zero subtracted from any integer is the integer itself.

e.g. $-6 - 0 = -6$

Worksheet 5

1. Write the negative of the following integers.

(a) -3 (b) 5 (c) 100

(d) -91 (e) 108 (f) -2004

2. Fill in the following blanks. The first one is done for you.

(a) $9 - 4 = 9 + \boxed{(-4)}$ (b) $12 - 7 = 12 + \boxed{}$

(c) $3 - (-2) = 3 + \boxed{}$ (d) $-4 - 6 = -4 + \boxed{}$

(e) $70 - (-19) = 70 + \boxed{}$ (f) $37 - 26 = 37 + \boxed{}$

(g) $-21 - 64 = -21 + \boxed{}$ (h) $0 - 8 = 0 + \boxed{}$

(i) $11 - (-6) = 11 + \boxed{}$ (j) $-100 - (-100) = -100 + \boxed{}$

3. Subtract the first integer from the second one.

(a) 9, 4 (b) – 9, 4

(c) 10, – 7 (d) – 11, – 6

(e) 16, 0 (f) 2001, 201

(g) 458, – 263 (h) 0, – 565

(i) – 823, – 232 (j) 41623, 26413

4. Subtract – 6 from 3 and 3 from – 6. Are the results same?

5. Sum of two integers is 48. If one of them is – 25, find the other.

6. Subtract the sum of 38 and – 49 from – 100.

7. **Compare.**

(a) (– 25) – (– 15) ◯ (– 25) + (– 15)

(b) 18 + (– 8) ◯ 18 – (– 8)

8. **Find the value of–**

(a) (– 3) – (– 19)

(b) – 12 – 8 – (– 35)

(c) 56 – (– 13) + 15

(d) (– 41) + (– 36) – 23

(e) (– 16) – (– 6) + (– 9) – 4

(f) 71 – 83 – (– 42) + 15

C. MULTIPLICATION OF INTEGERS

(i) Multiplication of two positive integers

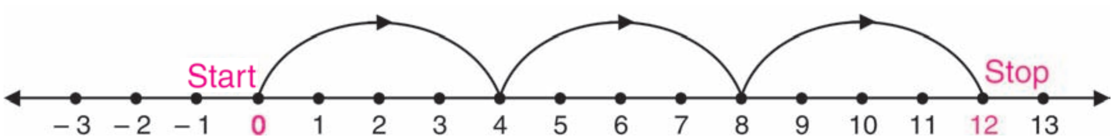
Let us multiply + 3 by + 4

(+ 3) × (+ 4) means + 4 is added 3 times

(+ 4) + (+ 4) + (+ 4) = + 12

> **Remember**
>
> Multiplication is repeated addition.

We take three jumps of 4 units each to the right.



So (+ 3) × (+ 4) = + 12

We reach at 12

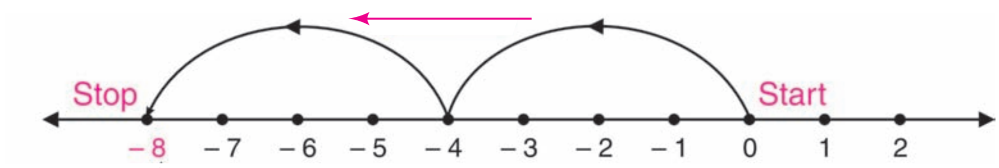
When both integers are positive, we multiply their absolute values and prefix plus sign to the product.

(ii) Multiplication of a positive and a negative integer

Let us multiply (+ 2) × (– 4)   – 4 is repeatedly added two times

(+ 2) × (– 4) = (– 4) + (– 4) = – 8

We take two jumps of 4 units each to the left.



We reach at – 8

So, (+ 2) × (– 4) = – 8

When one integer is positive and the other is negative, we multiply their absolute values and prefix minus sign to their product.

(iii) Multiplication of two negative integers

See the following pattern

$$
\begin{array}{rcrcrl}
(-3) & \times & 4 & = & -12 & \\
(-3) & \times & 3 & = & -\ 9 & \longrightarrow \boxed{(-12)+3} \\
(-3) & \times & 2 & = & -\ 6 & \longrightarrow \boxed{(-9)+3} \\
(-3) & \times & 1 & = & -\ 3 & \\
(-3) & \times & 0 & = & 0 & \\
(-3) & \times & (-1) & = & +\ 3 & \\
(-3) & \times & (-2) & = & +\ 6 & \longrightarrow \boxed{+3+3}
\end{array}
$$

The product increases by 3 at each stage

The multiplier decreases by one at each stage

Similarly, $(-3) \times (-3) = +9$ $\boxed{+6+3}$

When both integers are negative, we multiply their absolute values and prefix plus sign.

Note: The teacher should take a few more examples to show the pattern.

(iv) Product of more than three factors

Find the product of $(-2) \times 3 \times (-1) \times 5 \times (-5)$

$$= \overbrace{(-6)}^{-2\times 3} \times (-1) \times 5 \times (-5)$$

$$= \overbrace{6}^{(-6\times -1)} \times 5 \times (-5)$$

$$= \overbrace{30}^{6\times 5} \times (-5)$$

$$\mathbf{= -150}$$

In multiplication, if the number of negative integers is–

- odd, the product is **negative.**
- even, the product is **positive.**

PROPERTIES OF MULTIPLICATION

Property-1: Product of any two integers is also an integer.

We have, $(-2) \times (+4) = -8$ $\quad$ (-8 is also an integer)

Property-2: Product remains the same even if we change the order of integers.

We have, $5 \times (-3) = -15$

$(-3) \times 5 = -15$

Same

Order of integers is changed.

Property-3: Product remains the same even when we change the groupings of the integers.

Let us multiply $[2 \times (-10)] \times 3$ in two different ways.

Grouping is changed

$[2 \times (-10)] \times 3$
$= (-20) \times 3$
$= -60$

$2 \times [(-10) \times 3]$
$= 2 \times (-30)$
$= -60$

Product is same

Property-4: Product of an integer and zero is zero.

We have, $(-5) \times 0 = 0$

$(+19) \times 0 = 0$

Property-5: 1 multiplied by any integer is the integer itself.

We have, $(-9) \times 1 = -9$

$(+24) \times 1 = +24$

Note: One (1) is the identity element of multiplication.

Property-6: This property is called the distributive property of multiplication over addition.

If 2, (-3), 5 are three integers then,

$2 \times [(-3) + 5] = 2 \times (-3) + 2 \times 5$

We have
$2 \times [(-3) + 5]$
$= 2 \times 2$
$= 4$

We have
$2 \times (-3) + 2 \times 5$
$= (-6) + 10$
$= 4$

Same

Worksheet 6

1. **Write the appropriate sign of the product.**

(a) $(-3) \times (+5) = \boxed{}\ 15$

(b) $(+8) \times (-6) = \boxed{}\ 48$

(c) $(-15) \times (-3) = \boxed{}\ 45$

(d) $(+8) \times (-1) = \boxed{}\ 8$

(e) $(+9) \times (-9) = \boxed{}\ 81$

(f) $(-100) \times (-6) = \boxed{}\ 600$

(g) $(-11) \times (+11) = \boxed{}\ 121$

(h) $1000 \times (-100) = \boxed{}\ 100000$

2. **Find the product of the following:**

(a) $(-5) \times 6 =$ _______

(f) $(-25) \times 4 \times (-4) =$ _______

(b) $(-19) \times (-3) =$ _______

(g) $7 \times (-4) \times (-12) =$ _______

(c) $15 \times (-4) =$ _______

(h) $(-1) \times (-1) \times (-1) =$ _______

(d) $(-16) \times (-2) =$ _______

(i) $(-14) \times (-10) \times 6 \times (-1) =$ _______

(e) $(-5) \times 10 \times (-100) =$ _______

(j) $(-19) \times 7 \times 0 \times (-5) \times 2 =$ _______

3. **Find the value of the following:**

(a) $1234 \times 567 - 234 \times 567$

(b) $739 \times 99 - (-739)$

(c) $(-70) \times (10 - 5 - 22 - 83)$

(d) $861 \times (-3) + (-861) \times 7$

(e) $326 \times (-108) + 326 \times 8$

(f) $242 \times (-95) + 242 \times (-4) - 242$

4. **Write the integer which when multiplied by (– 1) gives,**

(a) – 3

(b) 19

(c) 0

(d) 74

(e) – 69

(f) – 100

5. **Compare the following:**

(a) $(7 + 6) \times 10 \bigcirc 7 + 6 \times 10$

(b) $(11 - 9) \times 8 \bigcirc 11 - 9 \times 8$

6. **What will be the sign of the product of the following:**

(a) 7 negative and 3 positive integers. $\boxed{}$

(b) 26 negative and 10 positive integers. $\boxed{}$

(c) 11 negative and 11 positive integers.

(d) (– 4) × (– 5) × (– 6) × ________ × (– 13).

(e) (– 12) × (– 13) × (– 14) × (– 15) × ________ × (– 22).

7. Write 'True' or 'Flase' for the following statements.

(a) The product of two integers is always an integer.

(b) The product of two integers with opposite signs is positive.

(c) The identity element of multiplication is 0.

(d) Of the two integers if one is negative, the product must be negative.

D. DIVISION OF INTEGERS

We know that every multiplication fact has two corresponding division facts.

We know, $4 \times 8 = 32$

$32 \div 8 = 4$ $\quad$ $32 \div 4 = 8$

Similarly, $(-3) \times (-9) = +27$

$27 \div (-9) = -3$ $\quad$ $27 \div (-3) = -9$

(i) Division of integers with like signs.

Divide + 20 by + 5

$(+20) \div (+5) = (+4)$

Like signs (+) $\quad$ Positive sign

Now, divide (– 12) by (– 3)

$(-12) \div (-3) = (+4)$

Like signs (–) $\quad$ Positive sign

Remember

Division is the inverse of multiplication.

To divide two integers of like signs, we divide their absolute values and prefix plus (+) sign.

(ii) Division of integers with unlike signs

Divide 6 by (– 3)

$(+6) \div (-3) = (-2)$ — negative sign

opposite sign

Now, divide 75 by (– 15)

$(+75) \div (-15) = (-5)$ — negative sign

opposite signs

> **To divide two integers of opposite signs, we divide their absolute values and prefix minus (–) sign.**

PROPERTIES OF DIVISION

Property-1: The quotient of two integers is not always an integer.

We have, $6 \div (-2) = -3$ (– 3 is an integer)

Is $2 \div (-3)$ an integer?

Is $(-6) \div 4$ an integer?

Property-2: When an integer (non-zero) is divided by the same integer, the quotient is one.

We have, $(-3) \div (-3) = 1$ $\quad$ $(+10) \div (+10) = 1$

Property-3: When an integer is divided by one, the quotient is the same integer.

We have, $(-7) \div 1 = -7$ $\quad$ $(+3) \div 1 = +3$

Property-4: Zero divided by any integer (non-zero) is zero.

We have, $0 \div (-9) = 0$ $\quad$ $0 \div (+3) = 0$

Worksheet 7

1. Put the appropriate sign in the quotients.

(a) $(-9) \div (+3) = \boxed{}\ 3$

(b) $(-30) \div (-10) = \boxed{}\ 3$

(c) $16 \div (-4) = \boxed{}\ 4$

(d) $(-21) \div (+3) = \boxed{}\ 7$

(e) $(-99) \div (-9) = \boxed{}\ 11$

(f) $(-105) \div (-7) = \boxed{}\ 15$

(g) $(+1000) \div (-100) = \boxed{}\ 10$

(h) $(+25) \div (-25) = \boxed{}\ 1$

2. Find the quotient of the following:

(a) $(-36) \div 9$ (b) $125 \div (-5)$

(c) $(-5375) \div (-25)$ (d) $374 \div (-17)$

(e) $(-108) \div 12$ (f) $0 \div (-17)$

(g) $(-3000) \div 100$ (h) $(-144) \div (-12)$

(i) $48 \div (-16)$ (j) $(-1331) \div (-11)$

3. Fill in the following blanks.

(a) $(-93) \div \boxed{} = (-93)$ (b) $17 \div (-1) = \boxed{}$

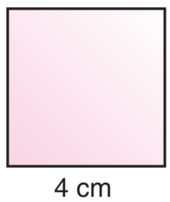
(c) $\boxed{} \div (-8) = 0$ (d) $\boxed{} \div 1 = -42$

(e) $(-65) \div \boxed{} = 1$

POWER OF INTEGERS

Remember

Area of a square = Side × Side
Volume of a cube = Edge × Edge × Edge

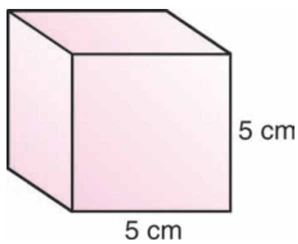


Now, let us look at the area of this square.

Area of square $= 4 \text{ cm} \times 4 \text{ cm} = 16 \text{ cm}^2$

4×4 can also be written as 4^2

So, $4^2 = 4 \times 4$



Now, look at the volume of this cube.

Volume of this cube $= 5 \times 5 \times 5 = 125 \text{ cm}^3$

$5 \times 5 \times 5$ can also be written as 5^3

or $5^3 = 5 \times 5 \times 5$

Similarly,

$2^4 = 2 \times 2 \times 2 \times 2$ $\rightarrow$ 2 is multiplied by itself four times

$(-10)^5 = (-10) \times (-10) \times (-10) \times (-10) \times (-10)$ $\rightarrow$ (– 10) is multiplied by itself five times

In 2^4, **2** is called the **Base** and **4** is called the **Power** or **Exponent**

Power or Exponent

In $\boxed{7^3}$

Base

Power or Exponent indicates the number of times the base is to be multiplied by itself.

We write	We read
2^2	Two square or two to the power two
6^3	Six cube or six to the power three
$(-7)^4$	Minus seven to the power four

See these examples.

Example 5: Find the value of

$(-2)^3 \times 5^2 \times (-10)^2$

Solution: We have,

$$(-2)^3 = (-2) \times (-2) \times (-2) = -8$$

$$5^2 = 5 \times 5 = 25$$

$$(-10)^2 = (-10) \times (-10) = 100$$

$$(-2)^3 \times 5^2 \times (10)^2 = (-8) \times 25 \times 100$$

$$= -20000$$

Remember

- (– 1) to the power of odd positive integer is equal to – 1.
- (– 1) to the power of even positive integer is equal to 1.

Example 6: Compute–

(a) $(-1)^3$

(b) $(-1)^6$

Solution: We have,

(a) $(-1)^3 = (-1) \times (-1) \times (-1) = \mathbf{-1}$

(b) $(-1)^6 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = \mathbf{1}$

Worksheet 8

1. Read aloud.

(a) 5^2 (b) 9^5 (c) 7^3

(d) $(-2)^4$ (e) $(-10)^3$ (f) $(-1)^{18}$

2. Complete the table given below. The first one is done for you.

Powered number	Base	Exponent
(a) 7^5	7	5
(b) 9^3		
(c) $(-3)^4$		
(d) $(-1)^6$		
(e) 20^2		
(f) $(-10)^7$		

3. Write in power notation.

(a) $4 \times 4 \times 4$ ☐

(b) $(-2) \times (-2) \times (-2) \times (-2)$ ☐

(c) $5 \times 5 \times 5 \times 5 \times 5$ ☐

(d) $(-10) \times (-10) \times (-10) \times$ .... 8 times ☐

(e) $(-11) \times (-11) \times (-11)$ ☐

(f) $(-1) \times (-1) \times (-1)$ .... 33 times ☐

4. Write the following in expanded form.

(a) 2^5

(b) 3^4

(c) $(-7)^3$

(d) $(-12)^2$

5. Compute the following:

(a) 3^4

(b) $(-5)^2$

(c) $(-1)^{78}$

(d) 11^3

(e) $(-4)^3 \times (-10)^3 \times (-1)^{789}$

(f) $(50)^2$

6. Find the number which is—

(a) Cube of – 9

(b) Square of 15

(c) 5th power of (– 10)

(d) 19th power of (– 1)

7. Simplify.

(a) $3^2 + 4^2$

(b) $2^3 - 4^2$

(c) $1^3 + 2^3 + 3^3$

(d) $(-10)^3 + (-10)^2 + (-10)^1$

(e) $3^3 - (-2)^3$

(f) $(-1)^{16} + (-1)^{36} + (-1)^7 + (-1)^{54}$

8. **Subtract the cube of (– 2) from the cube of 2.**

9. **Verify.**

(a) $(-2)^5 \times (-2)^3 = (-2)^8$ (b) $6^5 \times 6^4 = 6^9$

(c) $5^2 - 3^2 = 4^2$ (d) $12^2 + 5^2 = 13^2$

10. **Write 'True' or 'False' for the following statements.**

(a) $3^4 = 4^3$

(b) $9^7 \div 9^5 = 9^2$

(c) $(-5)^2 \times (-5)^3 \times (-5) = (-5)^6$

(d) $6^3 + 6^2 = 6^{3+2}$

(e) Cube of a negative integer is positive.

(f) $(-1)^{101} = -1$

(g) $1^3 = 3$

(h) Cube of a positive integer is negative.

(i) $3^2 = 6$

(j) 6th power of a negative integer is positive.

11. **What power of–**

(a) 2 is 32 (b) – 4 is – 64

(c) 10 is 100000 (d) (– 5) is – 125

VALUE BASED QUESTIONS

1. **Ravi and Rahul were good friends. Ravi was a poor boy. He was very much in need of a geometry box. Rahul decided to help him. He bought for him a geometry box costing ₹ 65 from his pocket money. Ravi was very excited to get the new geometry box and thanked Rahul for his caring nature.**

 (a) Express spending ₹ 65 as an integer.

 (b) Suggest any two ways by which you have helped any of your friends.

2. **In a quiz competition there were 25 questions. 2 marks was alotted to every correct answer and –1 to every wrong answer. Sheetal attempted 22 questions out of which 2 answers were wrong. The teacher gave her 40 marks. Sheetal went to the teacher and**

informed her that she has been given more marks. The teacher was happy with Sheetal. She did not deduct her marks.

(a) What is Sheetal's actual score?

(b) What quality of Sheetal made the teacher happy?

BRAIN TEASERS

1. **A. Tick (✓) the correct answer.**

(a) The number of integers between (– 10) and 3, is–

(i) 11 (ii) 13 (iii) 12 (iv) 14

(b) If we subtract (– 10) from (– 11) we get–

(i) – 1 (ii) 1 (iii) – 21 (iv) 21

(c) Square of 2 subtracted from cube of (– 1) is–

(i) 3 (ii) 5 (iii) – 5 (iv) 1

(d) Value of | – 7| + (– 6) + | 3 | is–

(i) – 10 (ii) 4 (iii) 10 (iv) – 4

(e) Which of the following does not lie to the right side of (– 61) on the number line?

(i) – 10 (ii) 18 (iii) – 49 (iv) – 73

B. Answer the following questions.

(a) Write any two integers less than (– 101).

(b) Find the value of | (– 30) – (– 7) |.

(c) Which integer added to (– 4) will give the integer 5?

(d) Simplify and write its opposite (– 3) × 5 × (– 1).

(e) Find the sum of the greatest negative integer and smallest positive integer.

2. **Indicate using integers.**

(a) 200 BC (b) 5° Celsius below zero

(c) Win by 3 goals (d) 40 km above sea level

3. **Write the opposites of the following statements.**

(a) India won the match by 3 wickets.

(b) Mohan withdrew ₹ 2500 from his bank account.

4. Write any three integers which are—

(a) smaller than – 25 (b) greater than – 191

5. Arrange in ascending order.

– 104, 48, – 69, 13, – 7, – 96, – 48, 5

6. Find the value on the number line.

(a) (– 3) + 5 – 7 (b) 8 + (– 6) + (– 2)

7. Simplify.

(a) (– 400) + 781 + (– 1400) + (– 81) + 300

(b) (– 273) + (– 541) + 900 + (– 511)

8. Subtract.

(a) – 9 from 0 (b) – 115 from – 115

9. Find the value of–

(a) (– 6) × [9 + (– 11)]

(b) 325 × (– 641) + 325 × (– 359)

(c) $5^2 \times (-1)^{19} \times (-2)^3 \times 3^2 \times (-10)^3$

10. Compare.

18 × (– 3) + 21 and 18 × [(– 3) + 21]

11. Fill in the blanks.

(a) There are ________ integers from – 4 to 11?

(b) Natural numbers are called ________ integers (positive/negative).

(c) The additive inverse of 6 is ________

(d) 168 + ________ = 0

(e) The predecessor of – 249 is ________

(f) [(– 2) + (– 7)] × 3 = 3 × ________ + 3 × ________

(g) The opposite of (– 3) × 2 × (– 1) is ________

(h) All the negative integers are ________ than zero.

(i) 14, 7, 0, – 7, ________, ________.

12. **Write 'True' or 'False' for the following statements.**

(a) The absolute value of an integer is always greater than the integer.

(b) The product of 9 negative integers is positive.

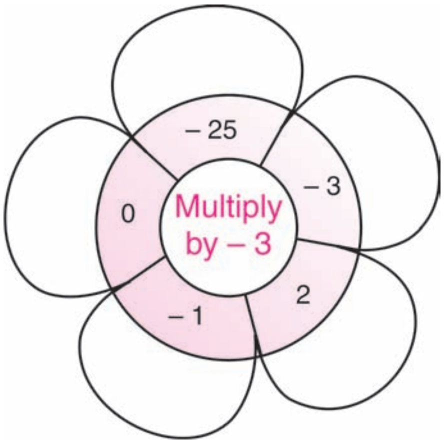
(c) Cube of 11 has 1 in its units place.

(d) The base in 7^3 is 3.

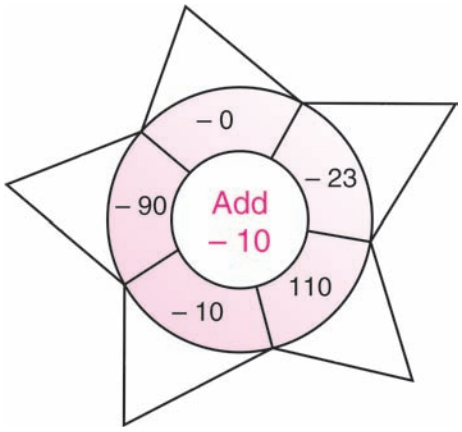
(e) $3^8 \div 3^5 = 3^3$

13. **Fill in the missing places with proper integers.**

(a)



(b)



HOTS

1. (a) **Calculate 1 – 2 + 3 – 4 + 5 – 6 + ... + 179 – 180.**

(b) **Find the value of 5 + (– 5) + 5 + (– 5) + 5 + ... if the number of fives are–**

(i) 148

(ii) 191

2. **A cement company gains ₹ 12 per bag of white cement sold and gets a loss of ₹ 8 per bag of grey cement sold.**

(a) If the company sells 3500 bags of white cement and 5000 bags of grey cement in a month, find the gain or loss.

(b) If the number of grey cement bags sold is 6000, how many bags of white cement should the company sell to have neither gain or loss?

YOU MUST KNOW

1. We need to use numbers with negative signs in some situations. These are called negative numbers. Some examples of their use are temperature of a day, water level in a sea, etc.
2. Positive numbers, negative numbers along with zero are called integers. Zero is neither positive or negative.
3. Each and every integer can be represented on the number line. The integer to the right side of another integer is greater.
4. Absolute value of an integer is the numerical value without taking the sign to account.
5. To add two positive integers or two negative integers, add their absolute values and prefix the sign of addends to it.
6. If integers are of opposite signs, we find the difference of their absolute values and prefix the sign of the integer whose absolute value is greater.
7. To subtract two integers, we add the negative of the subtrahend to the minuend.
8. In multiplication, if both integers have like signs we multiply their absolute values and prefix plus sign to the product and if the integer have unlike signs we multiply their absolute values and prefix negative sign to the product.
9. One is the identity element of multiplication of integers.
10. To divide two integers of like signs, we divide their absolute values and prefix (+) sign.
11. To divide two integers of unlike signs, we divide their absolute values and prefix (–) sign.
12. In 7^3, 7 is called the base and 3 is called exponent or power. Power or exponent indicates the number of times the base is to be multiplied by itself.